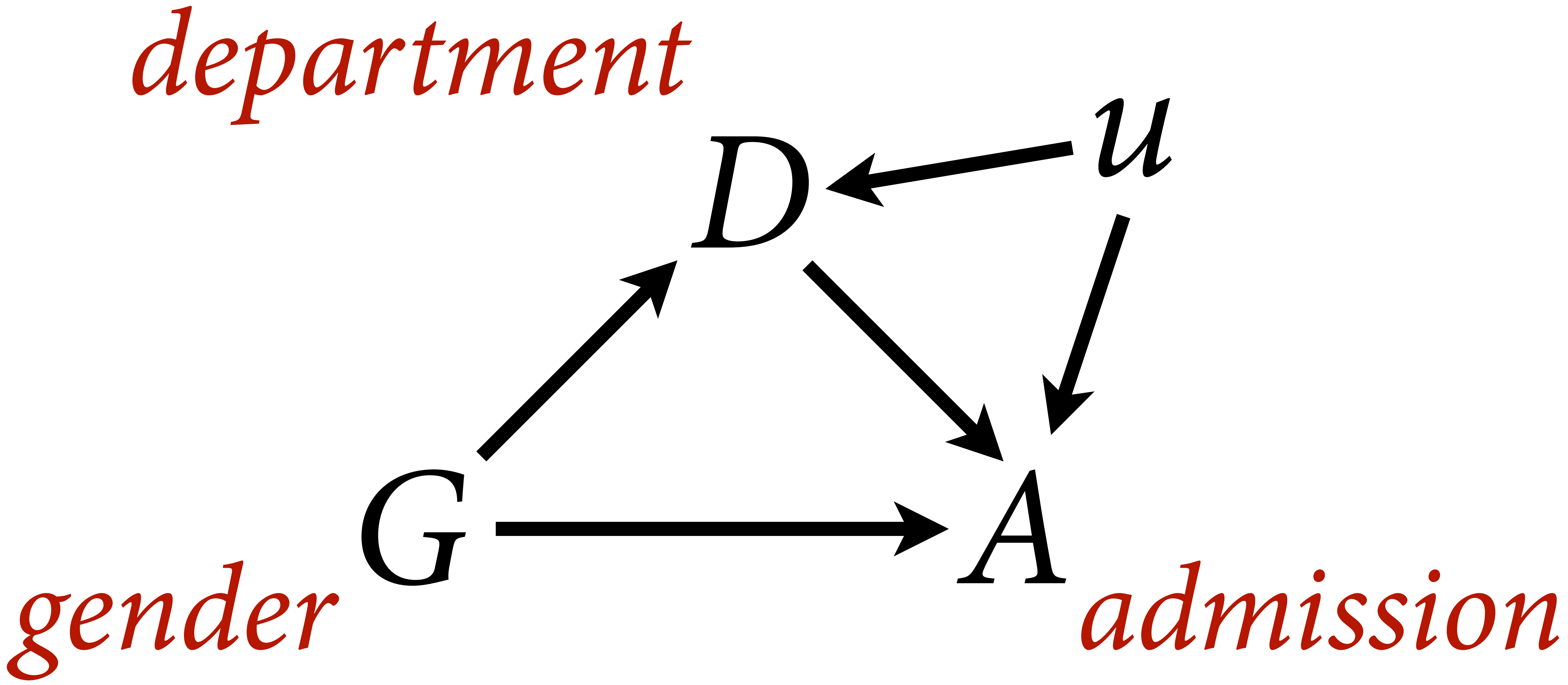
Statistical Rethinking

2022

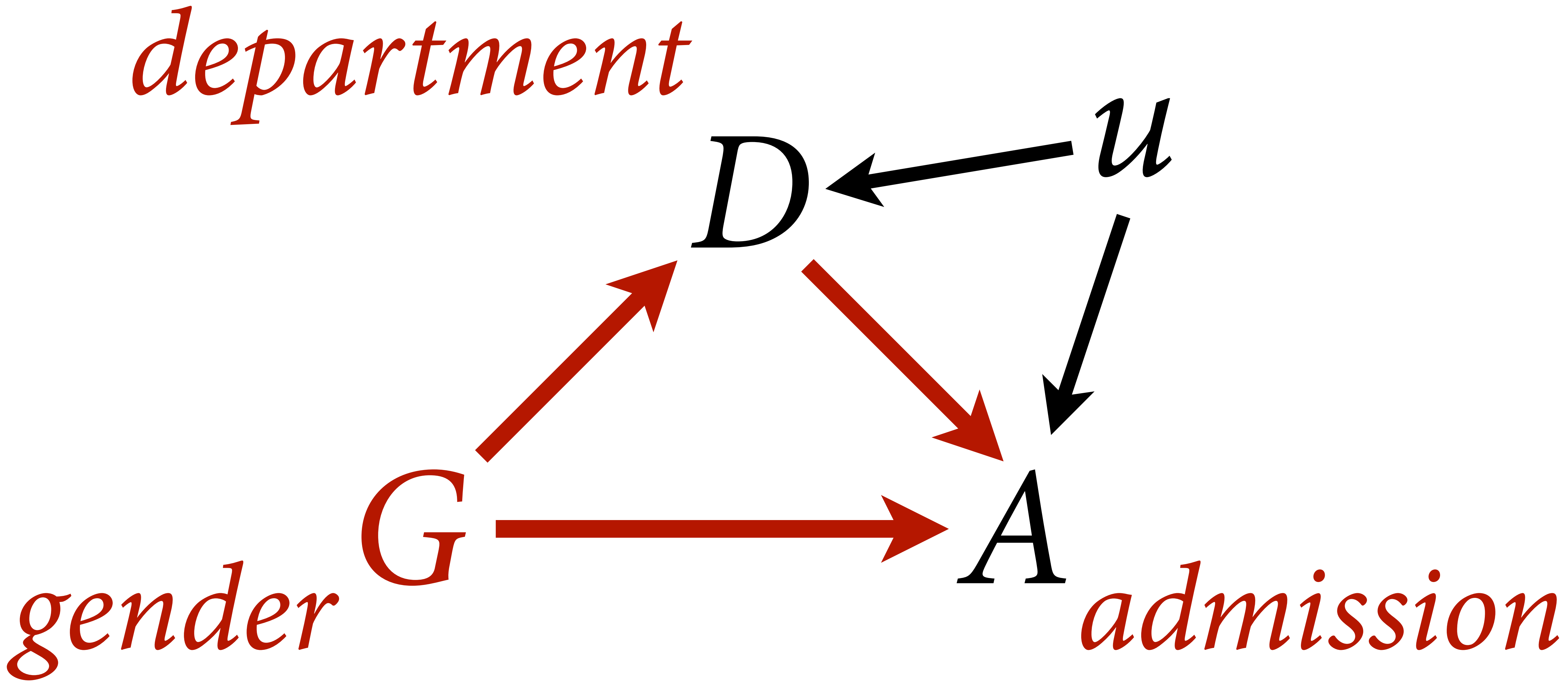
11: Ordered Categories

Key & Peele - Strike Force Eagle 3: The Reckoning

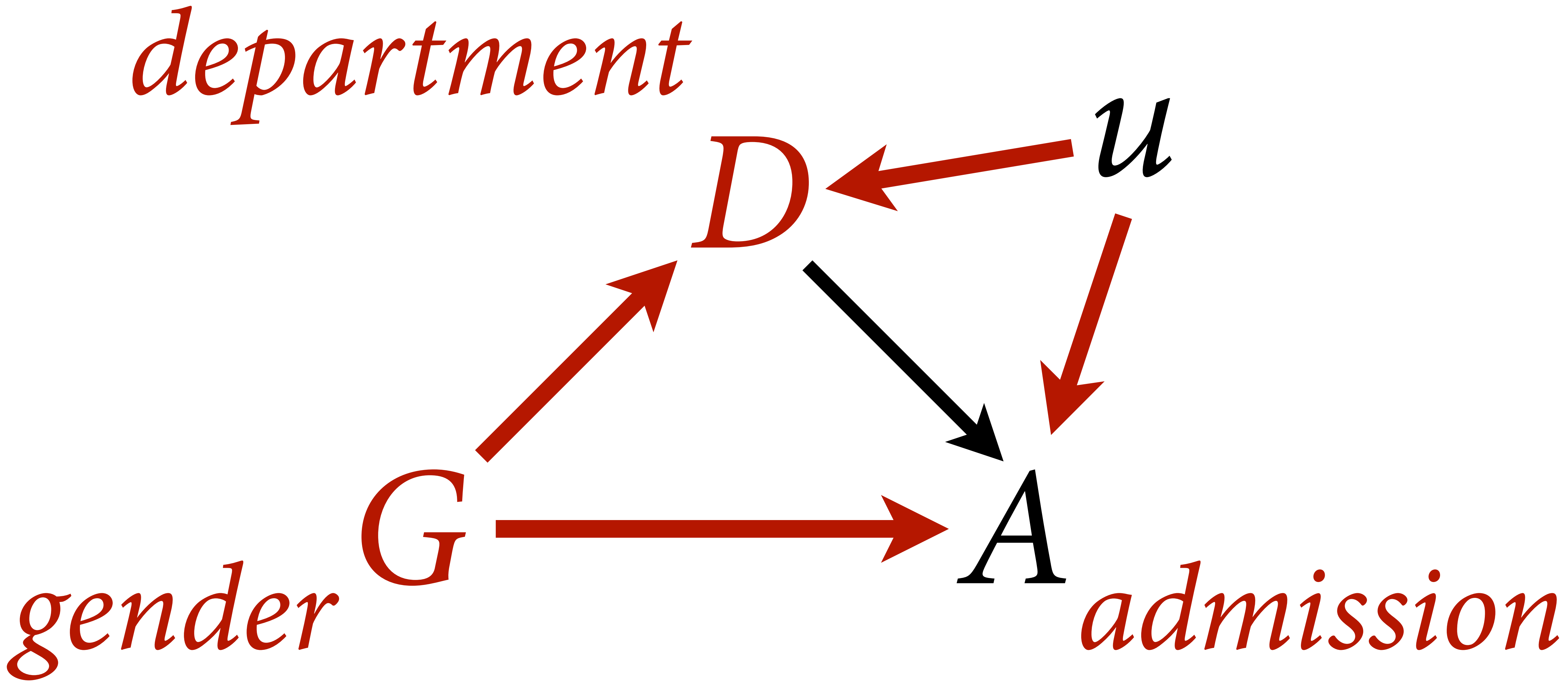
Can't always get what you want

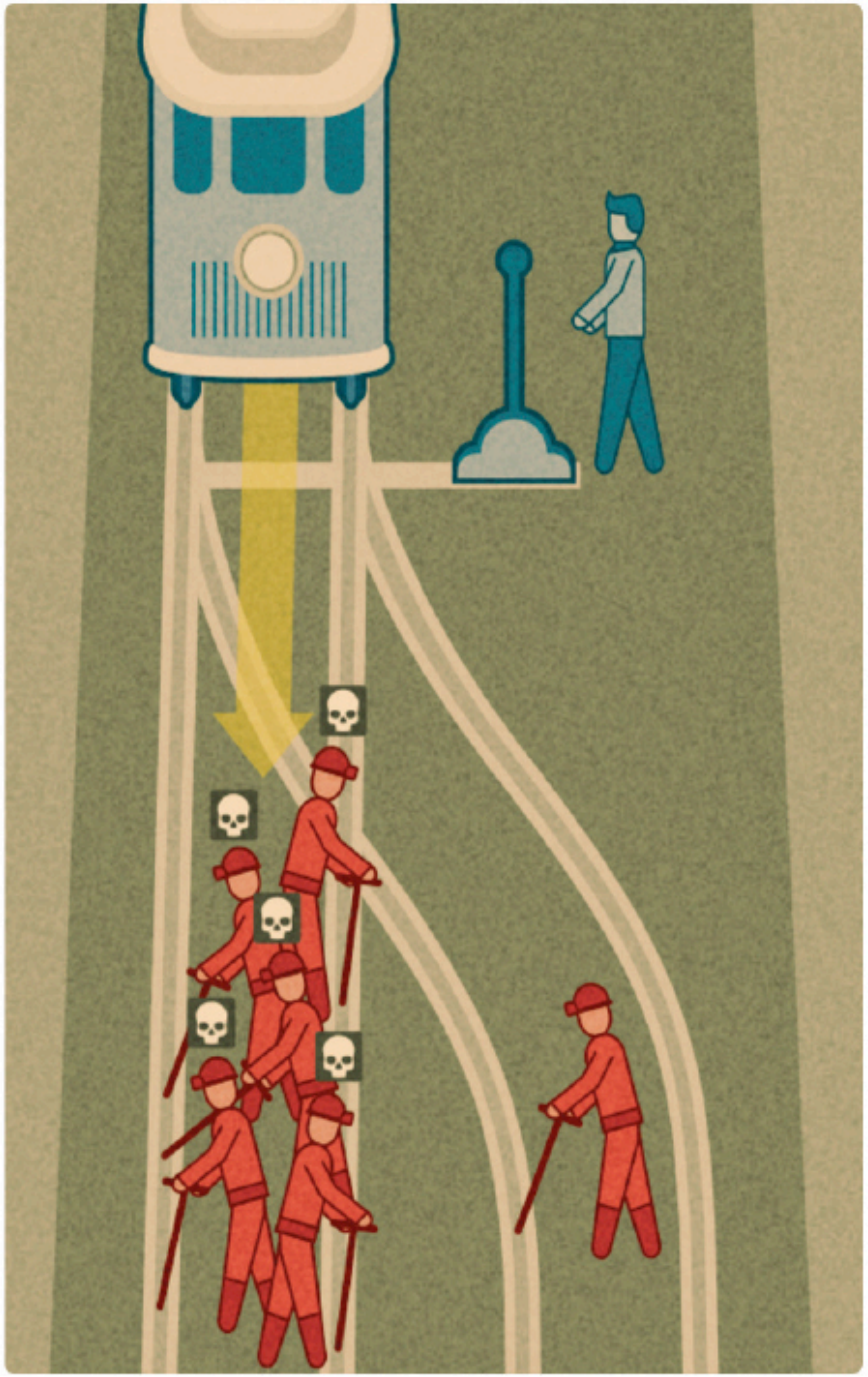


Can't always get what you want

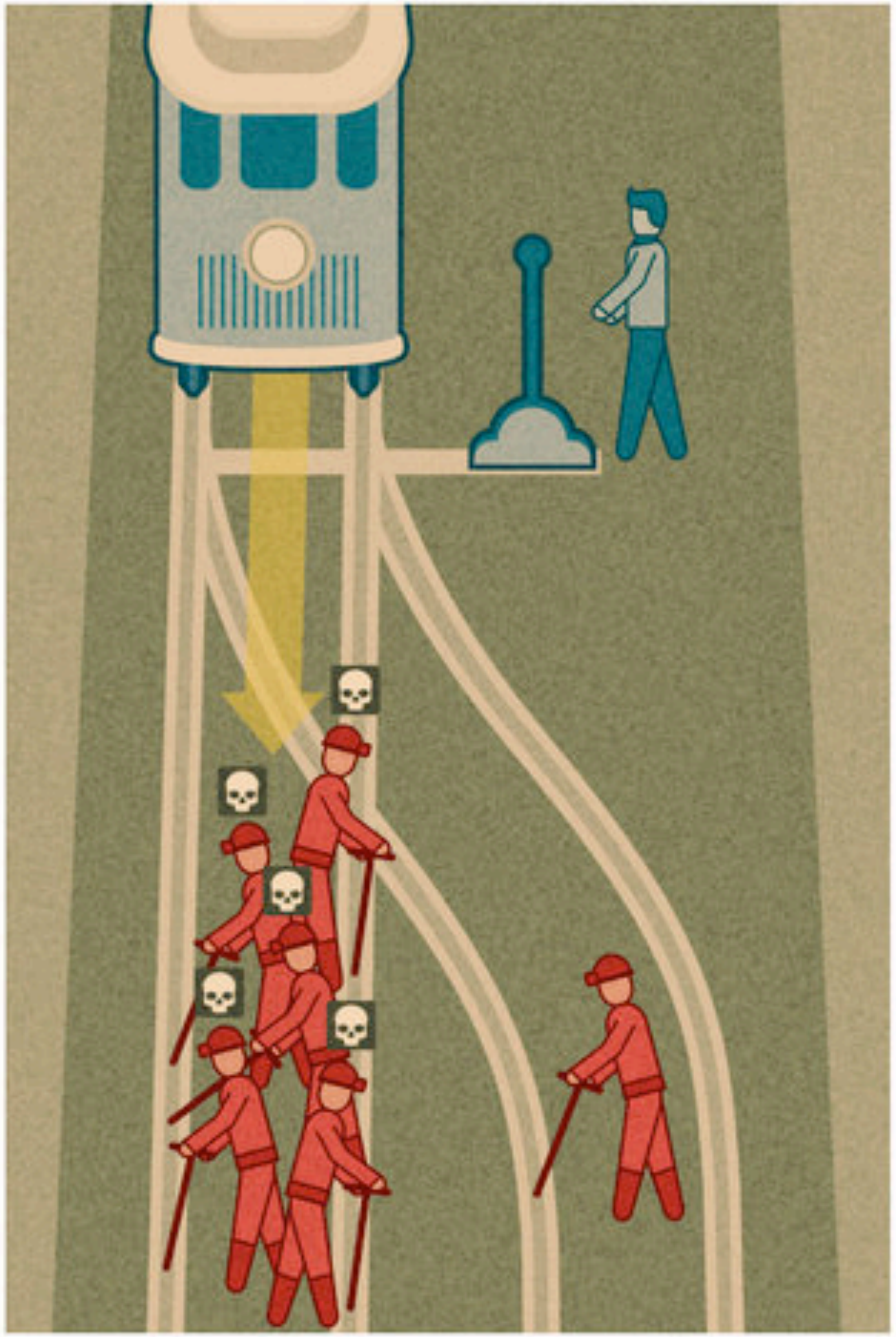


Can't always get what you want





Action



Trolley Problems

data(Trolley)

331 individuals (age, gender, edu)

Voluntary participation (online)

30 different trolley problems

action / intention / contact

9930 responses:
How appropriate (from 1 to 7)?

Trolley Problems

data(Trolley)

331 individuals (age, gender, edu)

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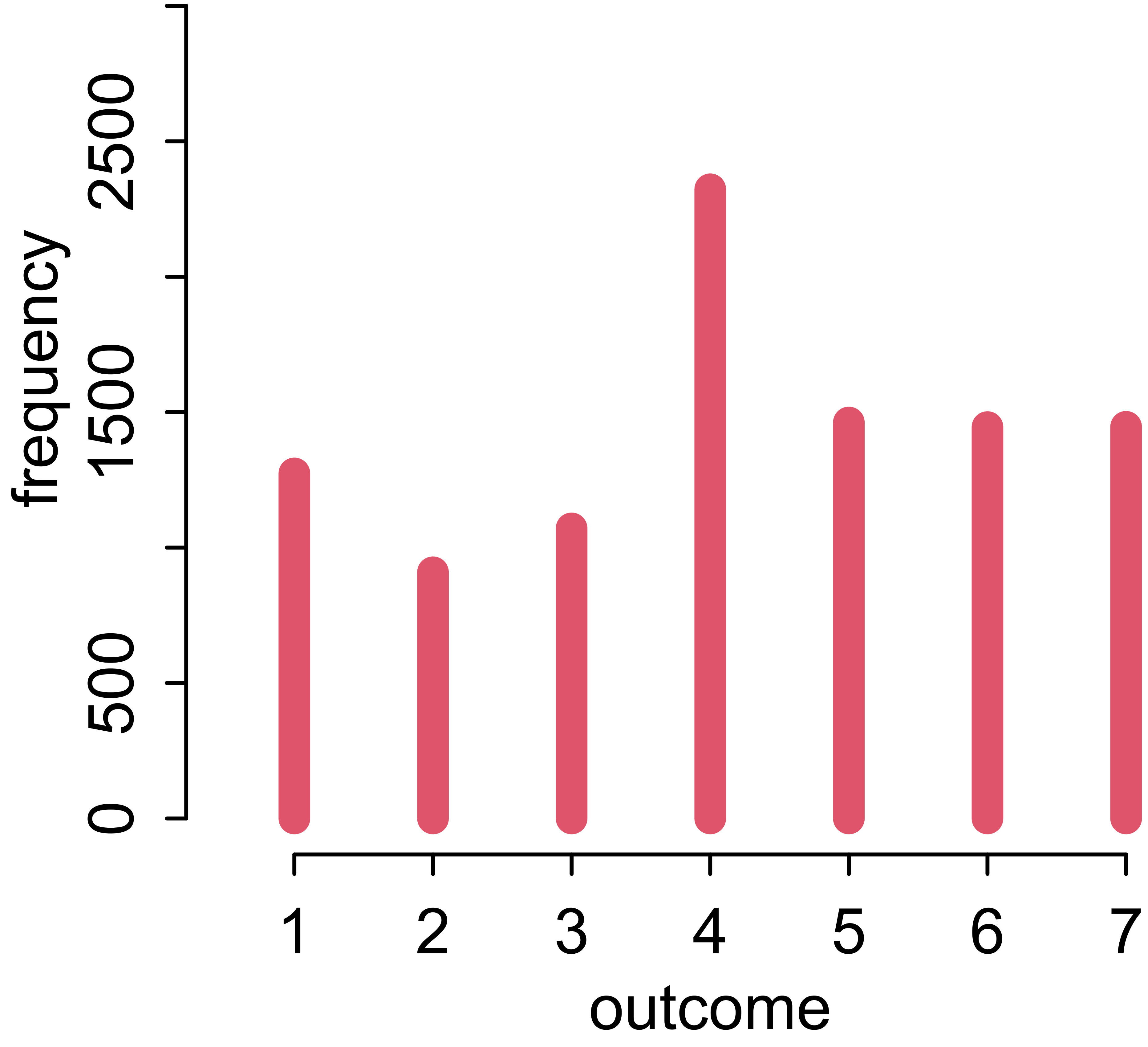
30 different trolley problems

action / intention / contact

9930 responses:
How appropriate (from 1 to 7)?

Ordered categorical

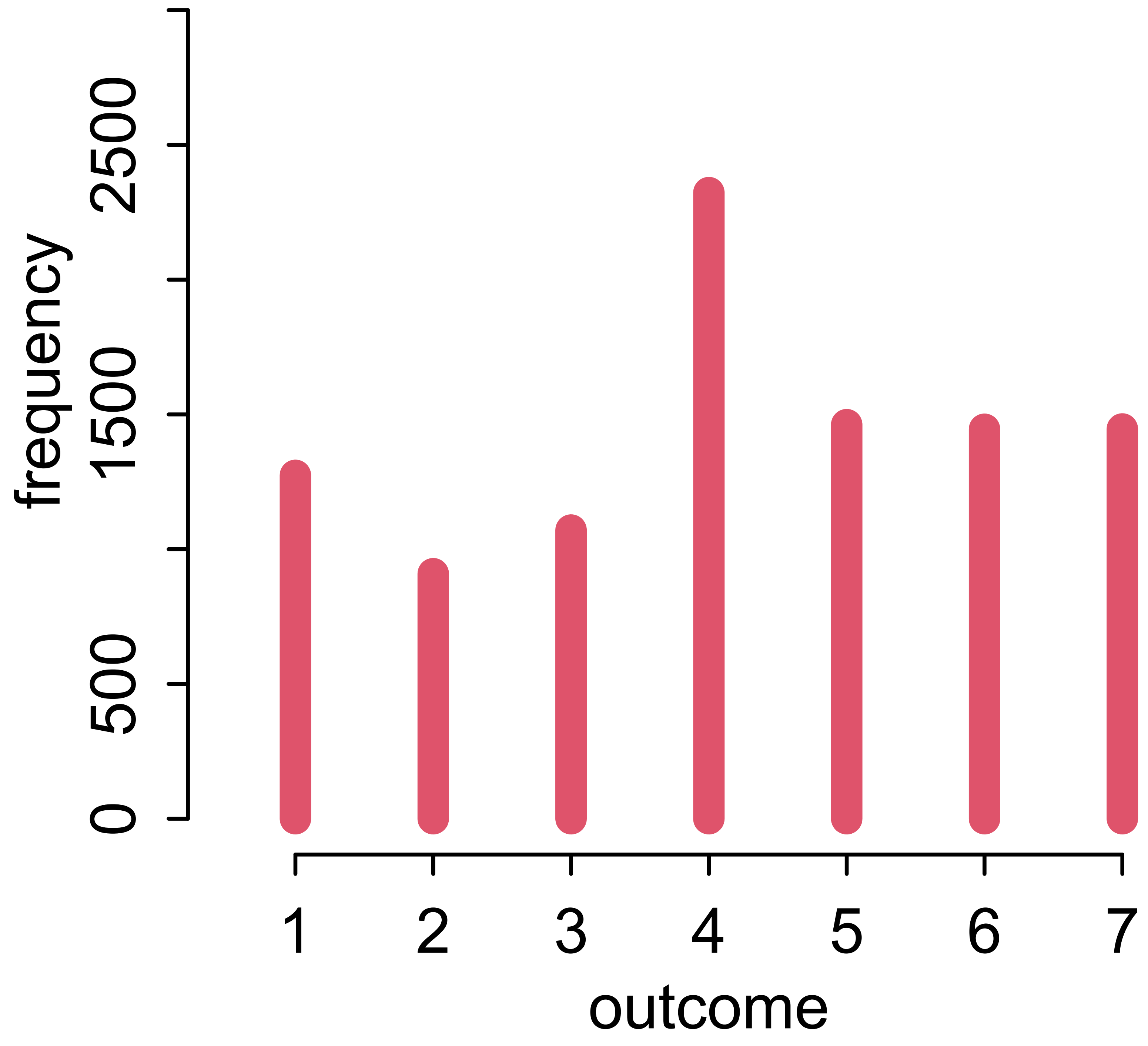


Estimand: How do **action**, **intention**, **contact** influence **response** to a trolley story?

treatment

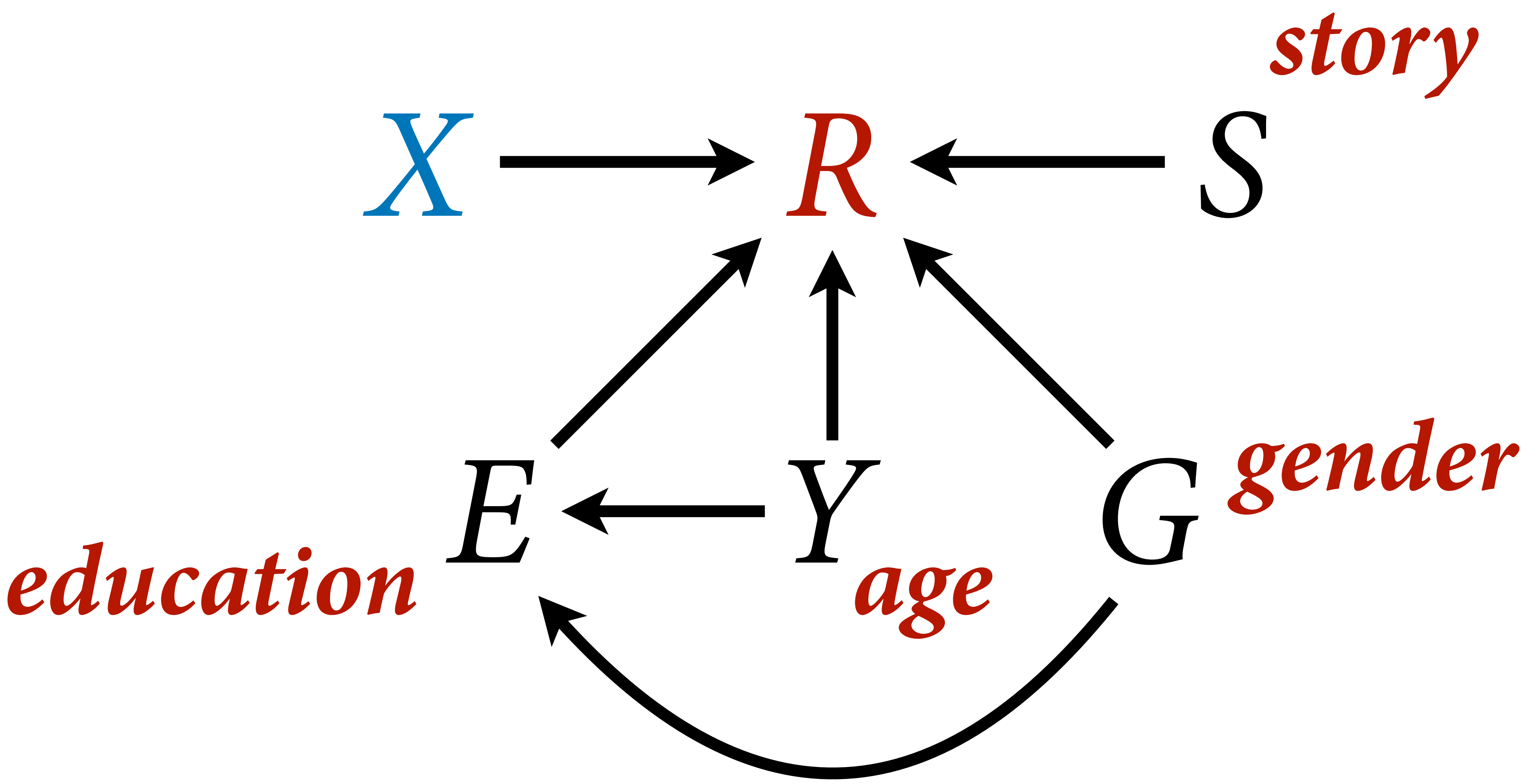
$X \longrightarrow R$

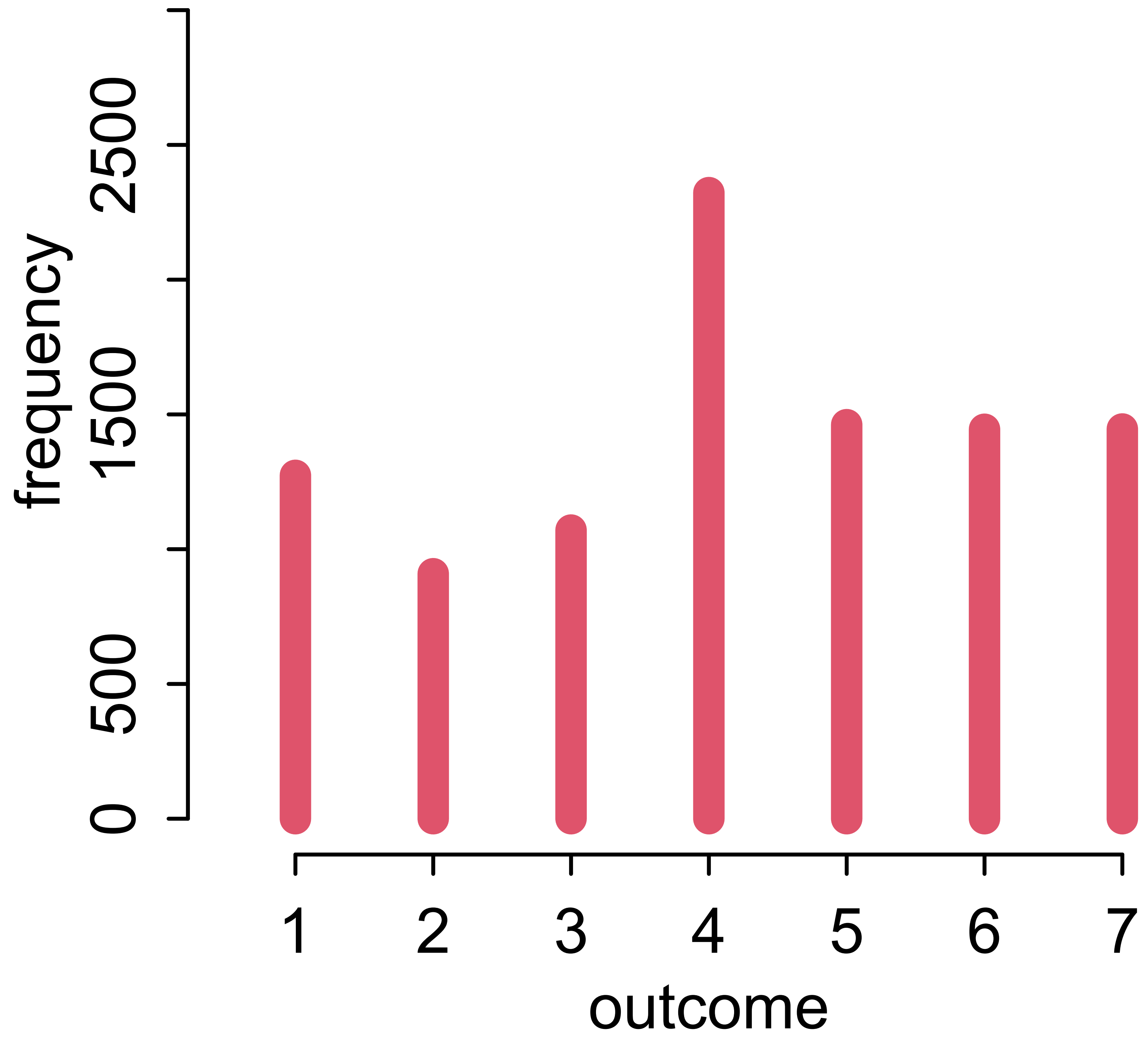
response



Estimand: How do **action**, **intention**, **contact** influence **response** to a trolley story?

How are influences of A/I/C associated with other variables?





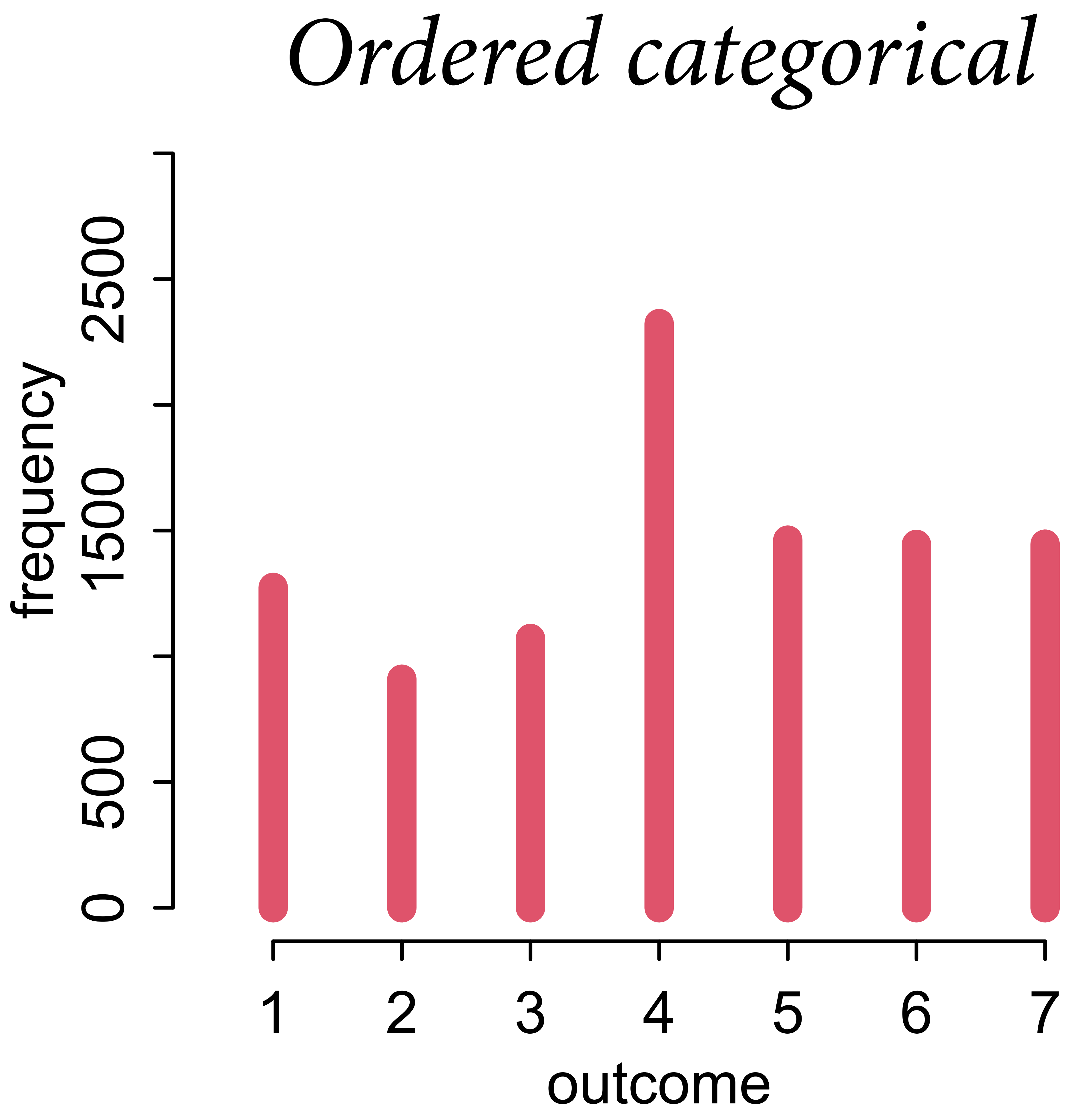
Ordered categories

Categories: Discrete types

cat, dog, chicken

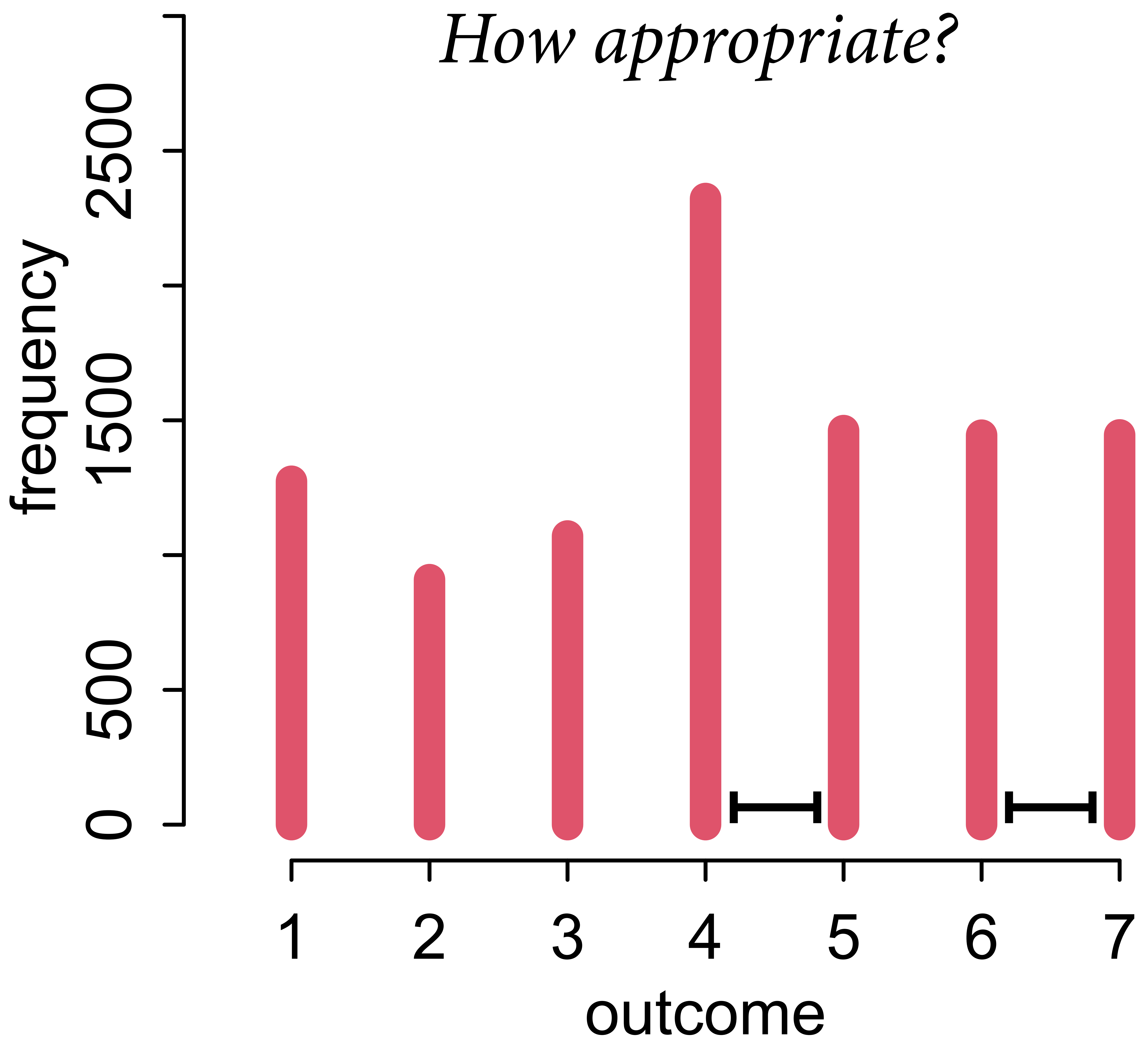
Ordered categories: Discrete types with ordered relationships

bad, good, excellent



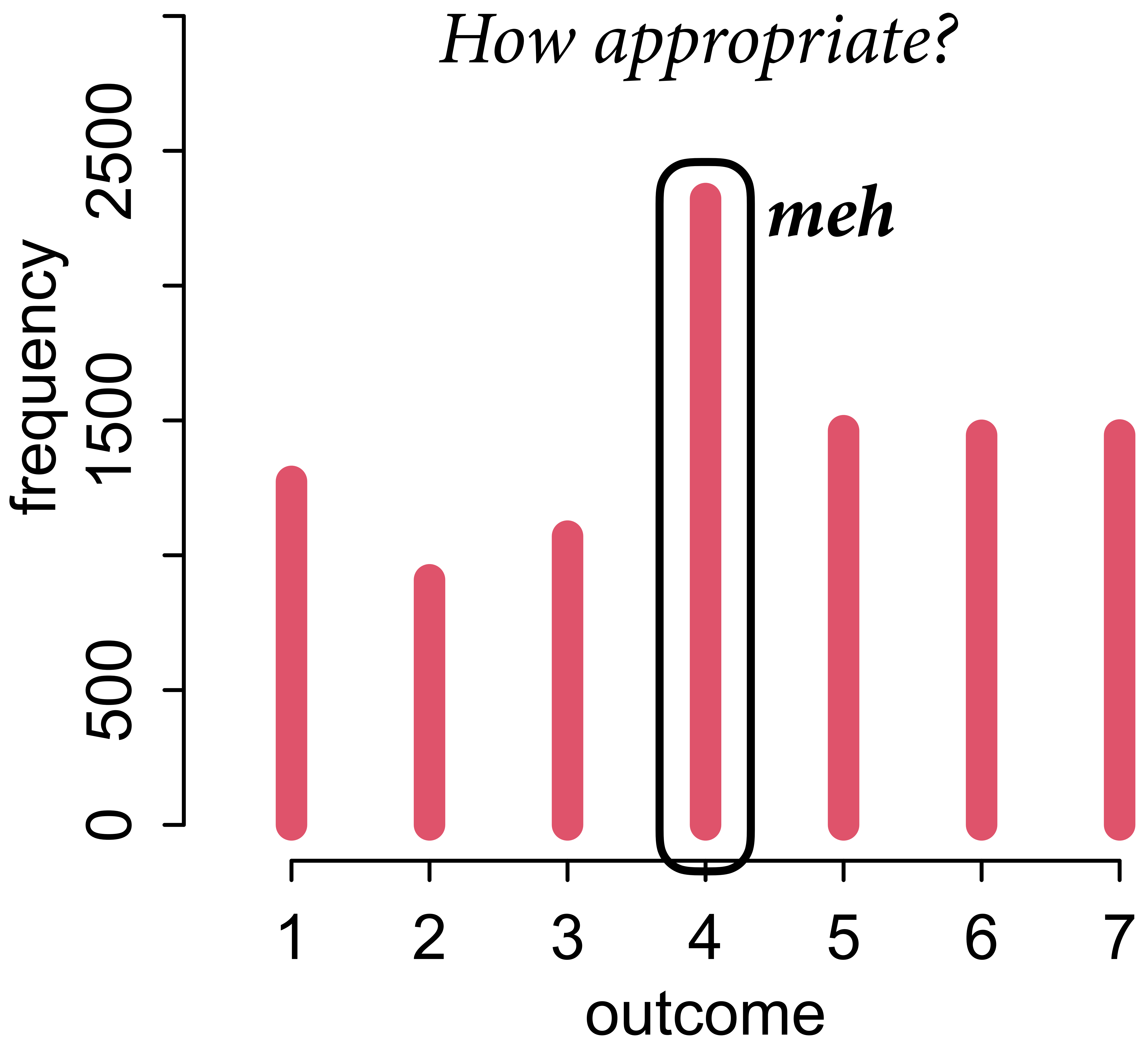
Distance between values not constant

Probably much easier to go from 4 to 5 than from 6 to 7

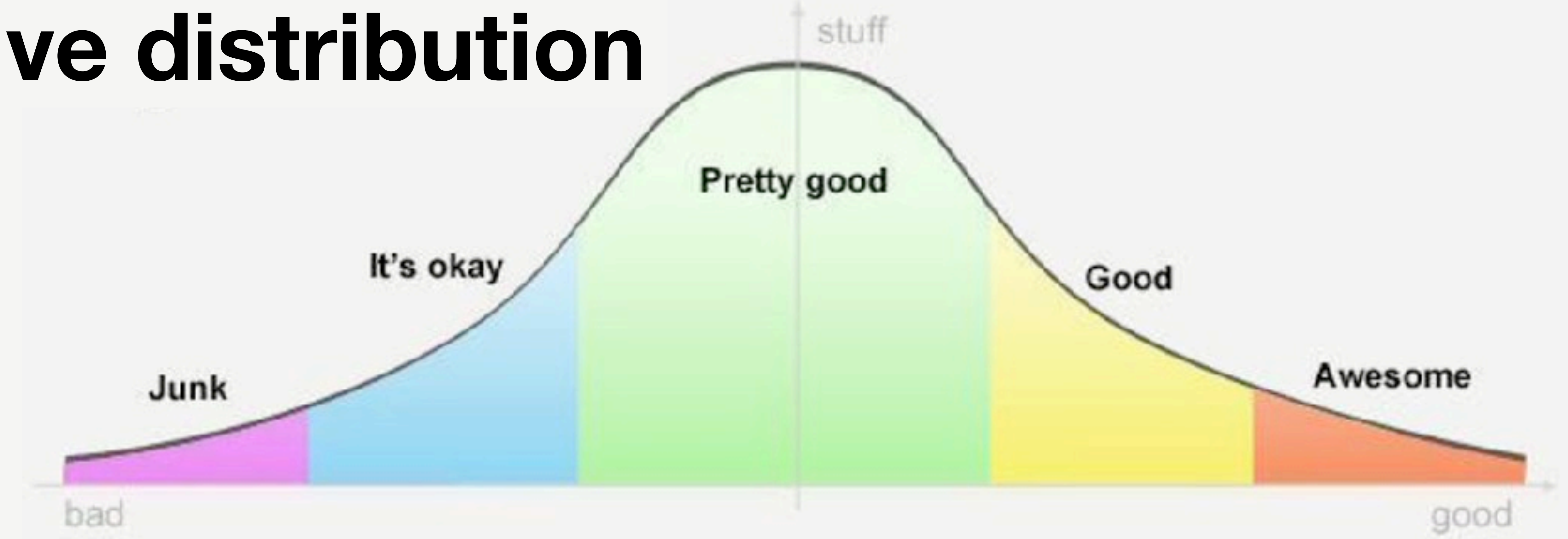


Anchor points common

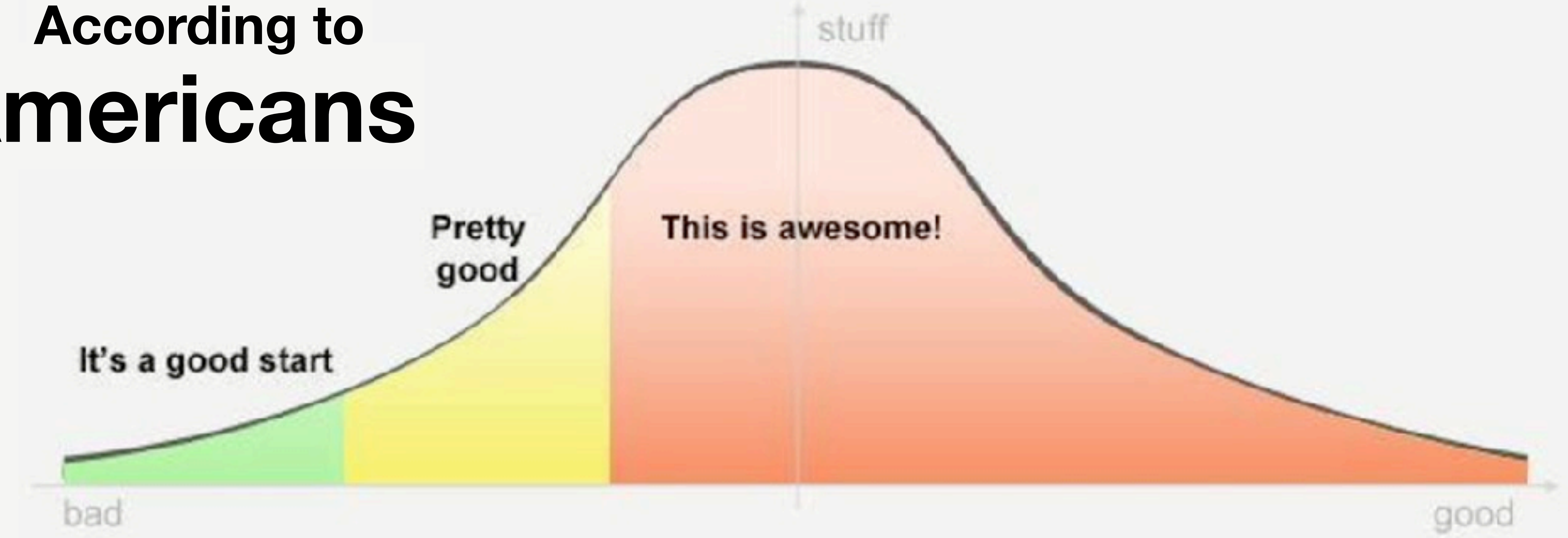
Not everyone shares the same anchor points

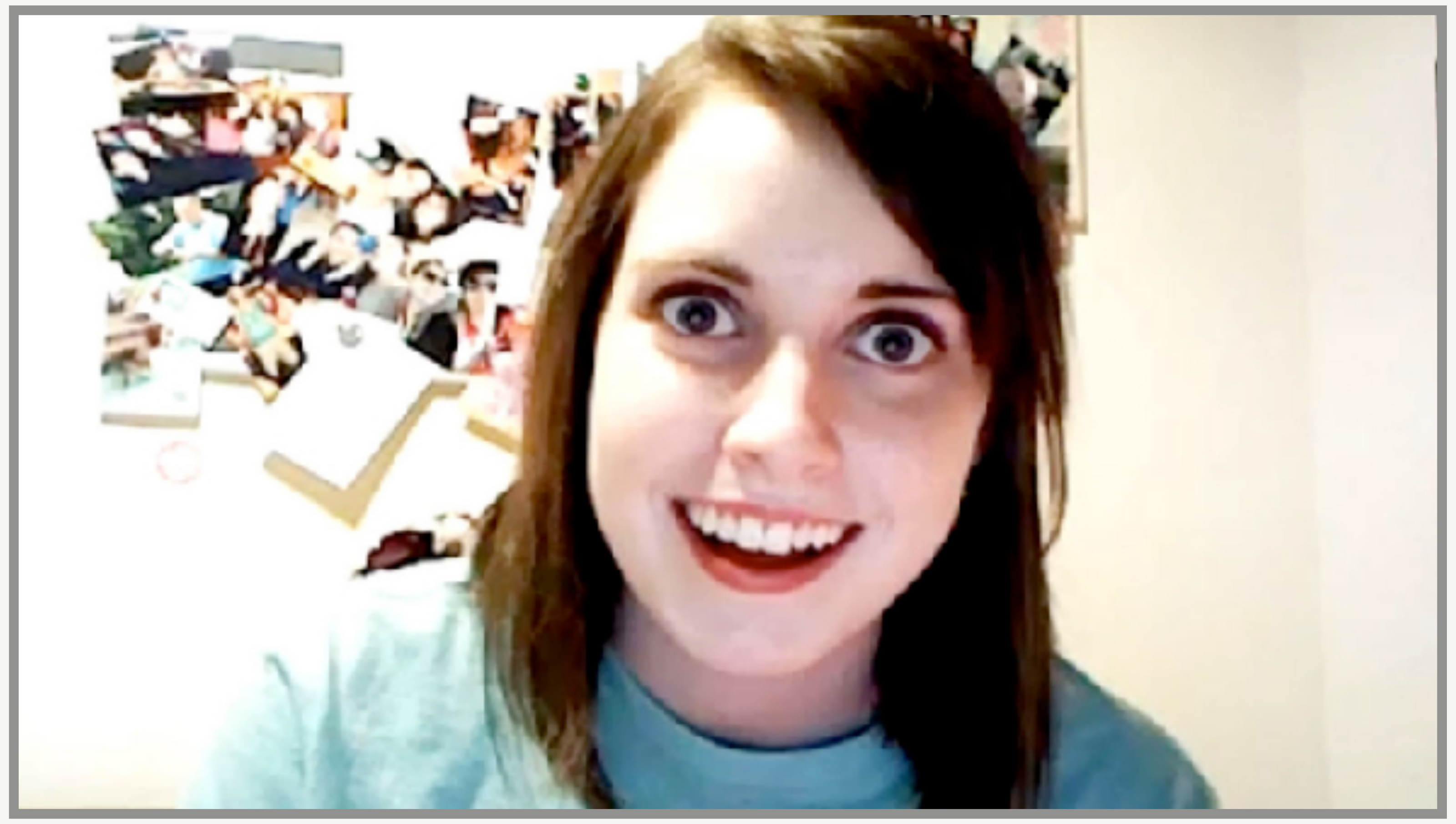


Objective distribution

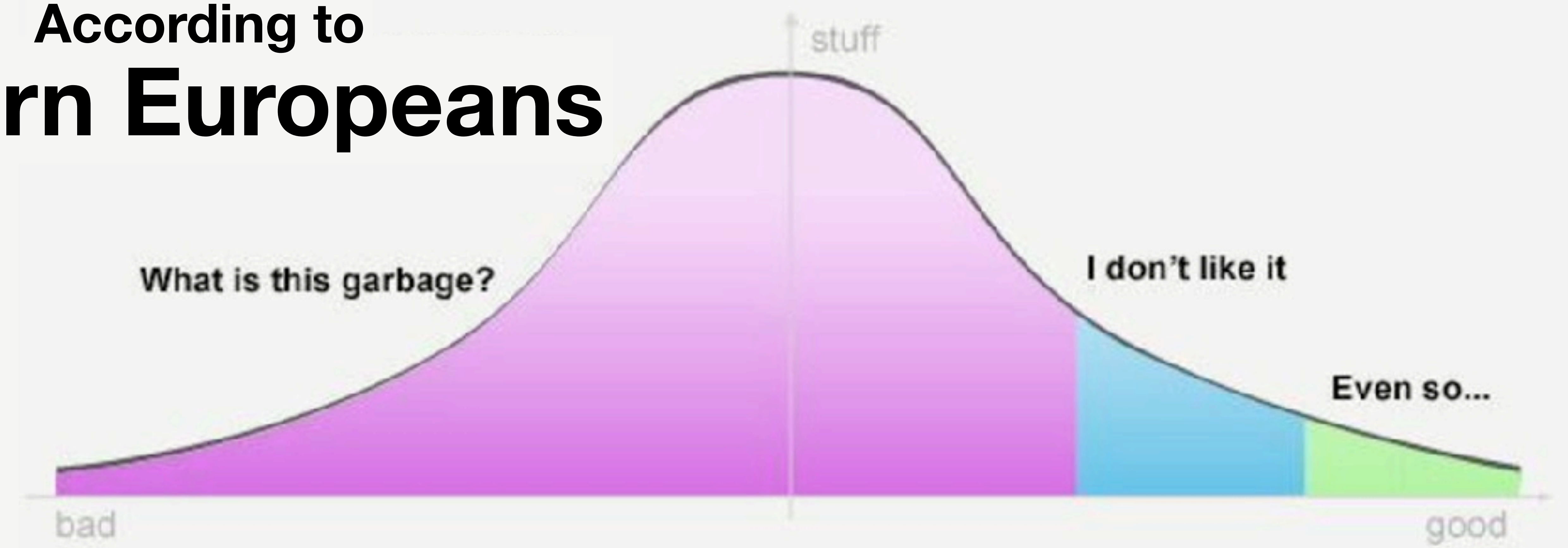


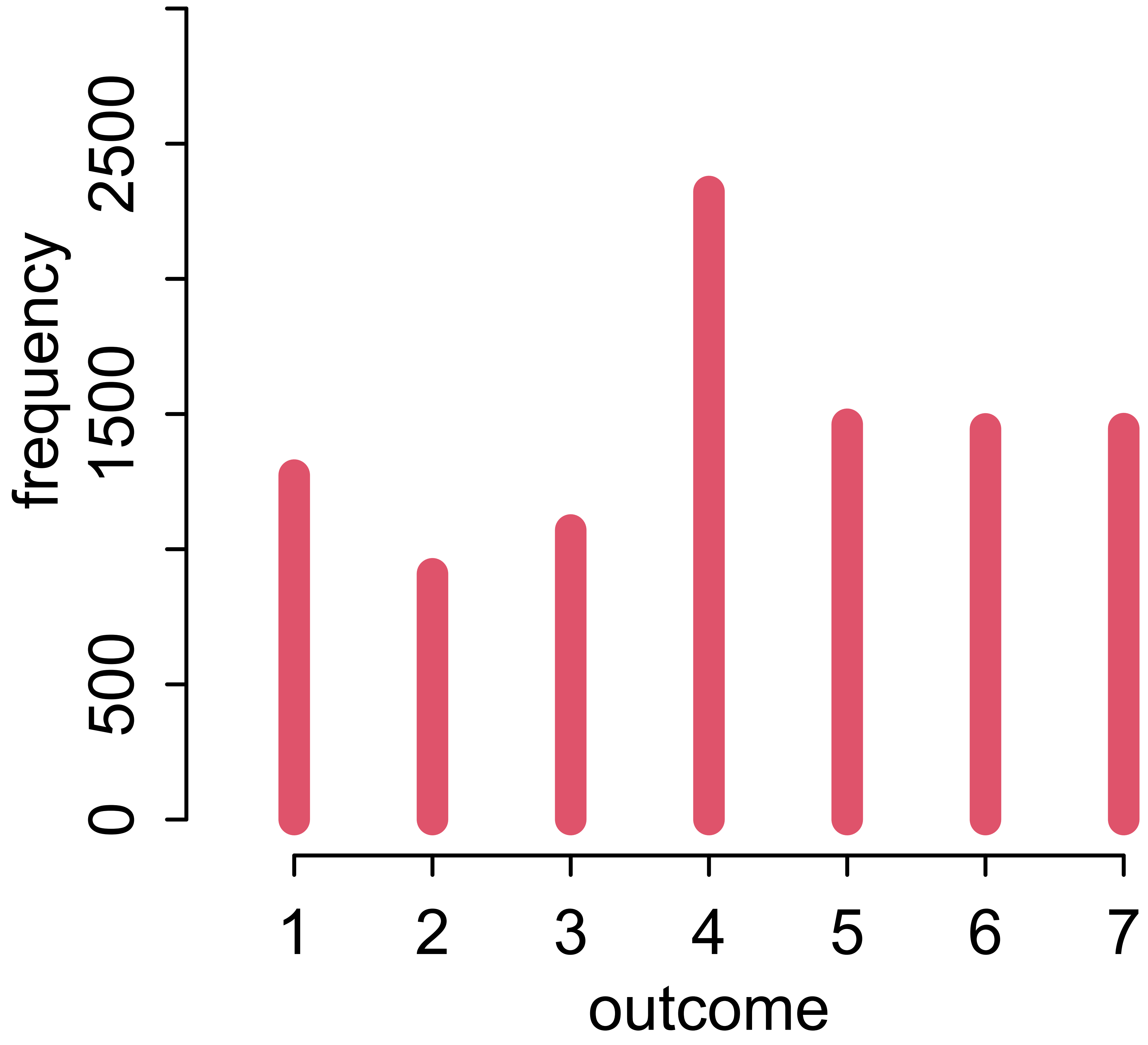
According to
Americans

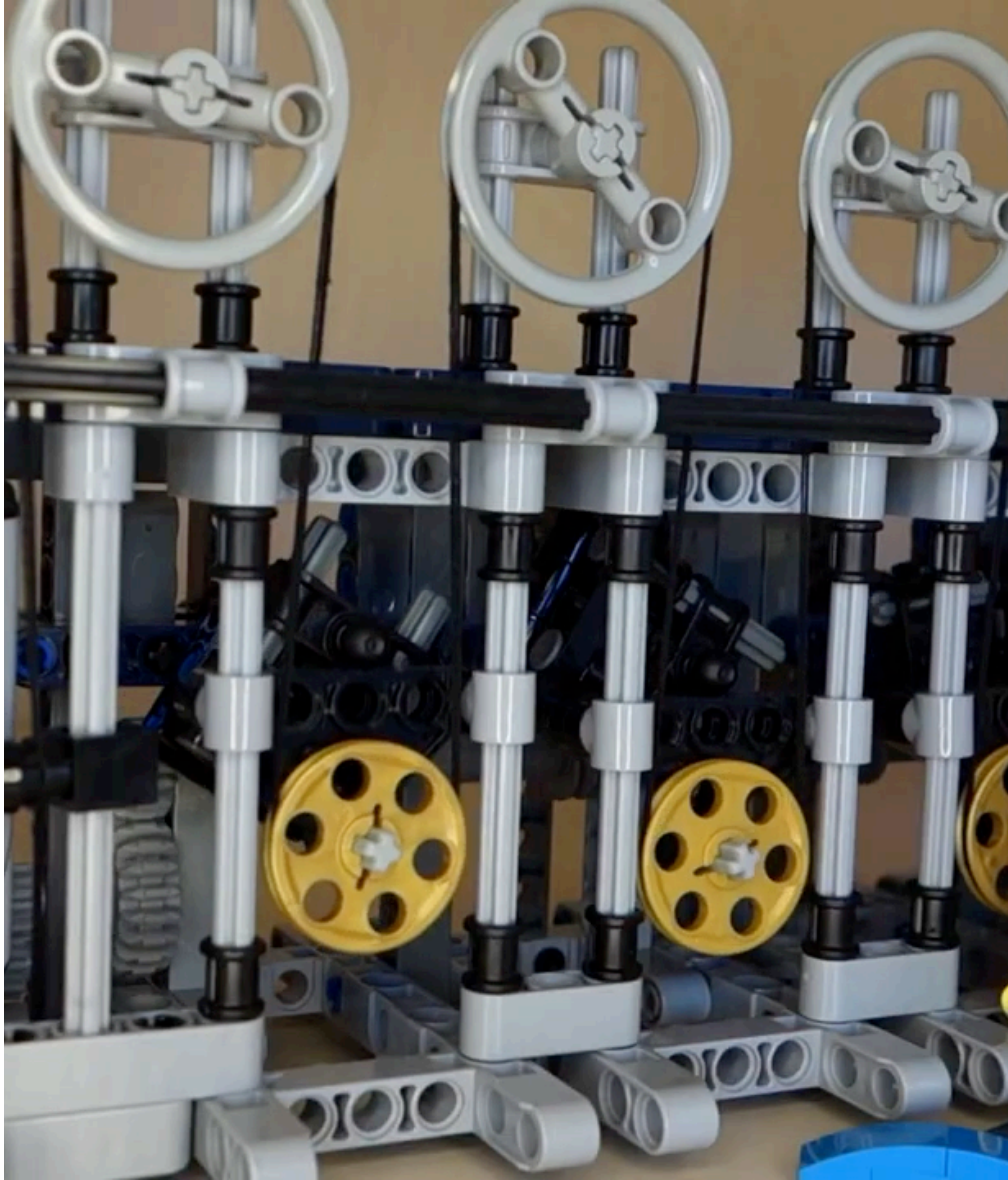




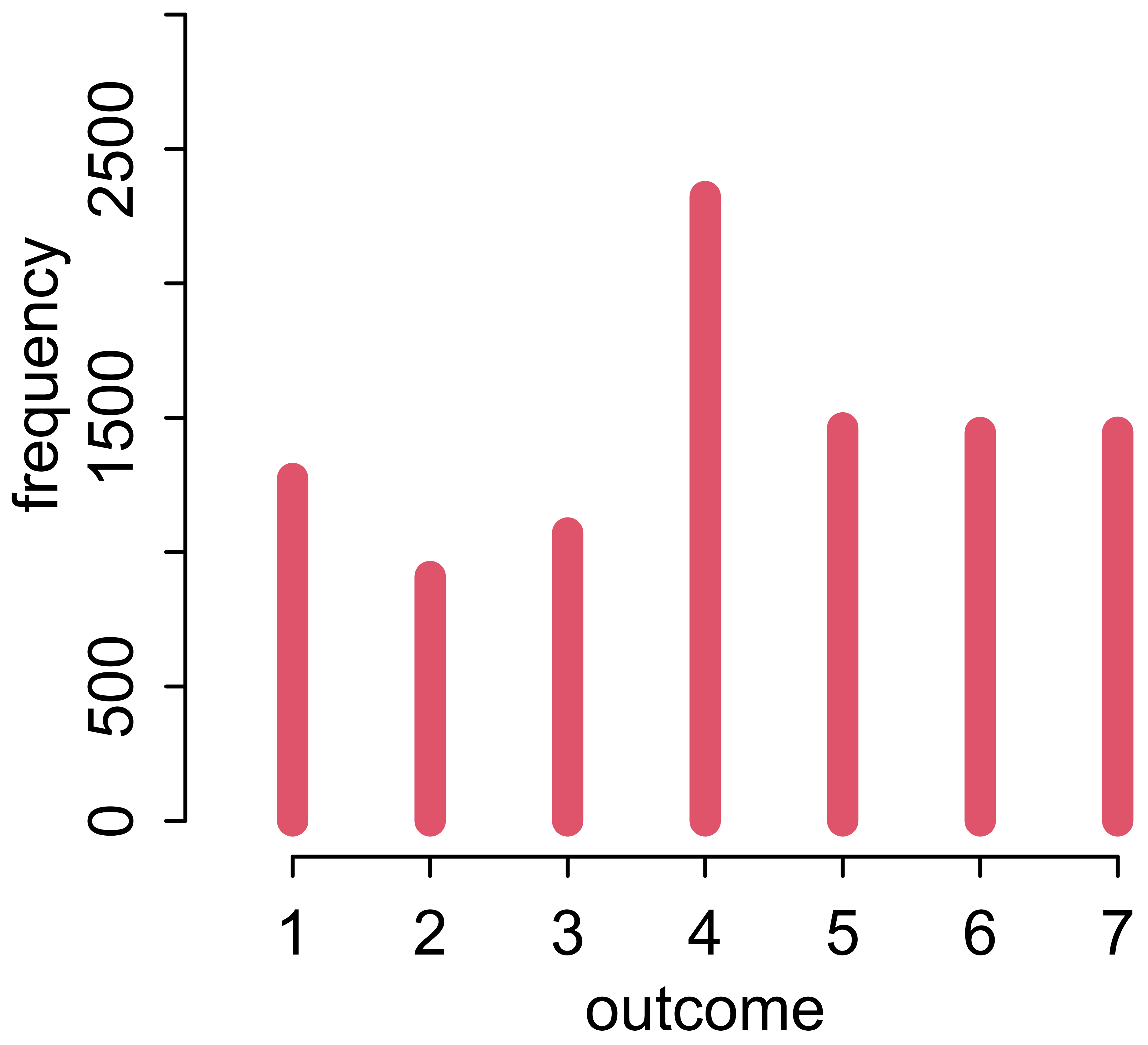
According to
Eastern Europeans

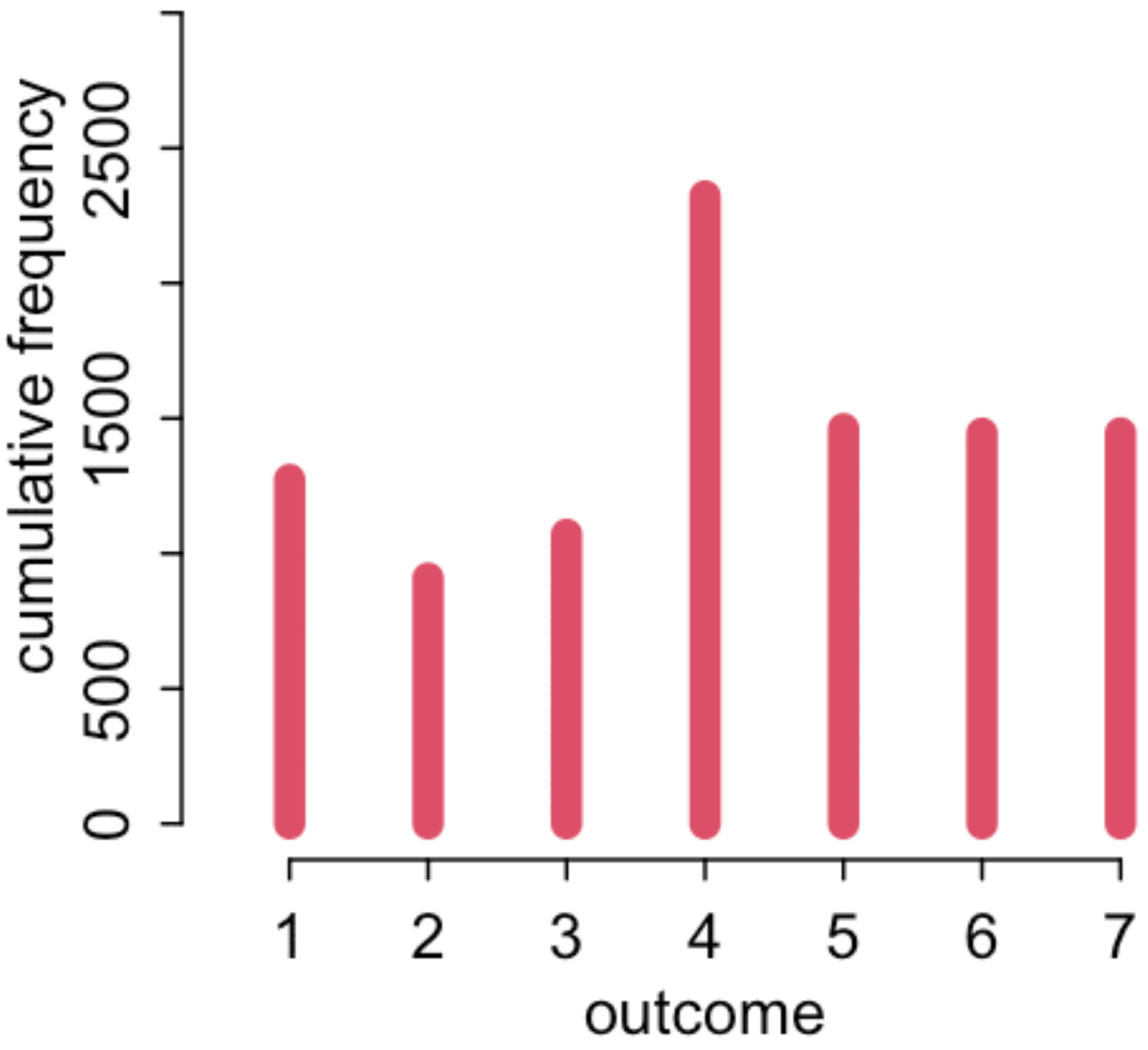


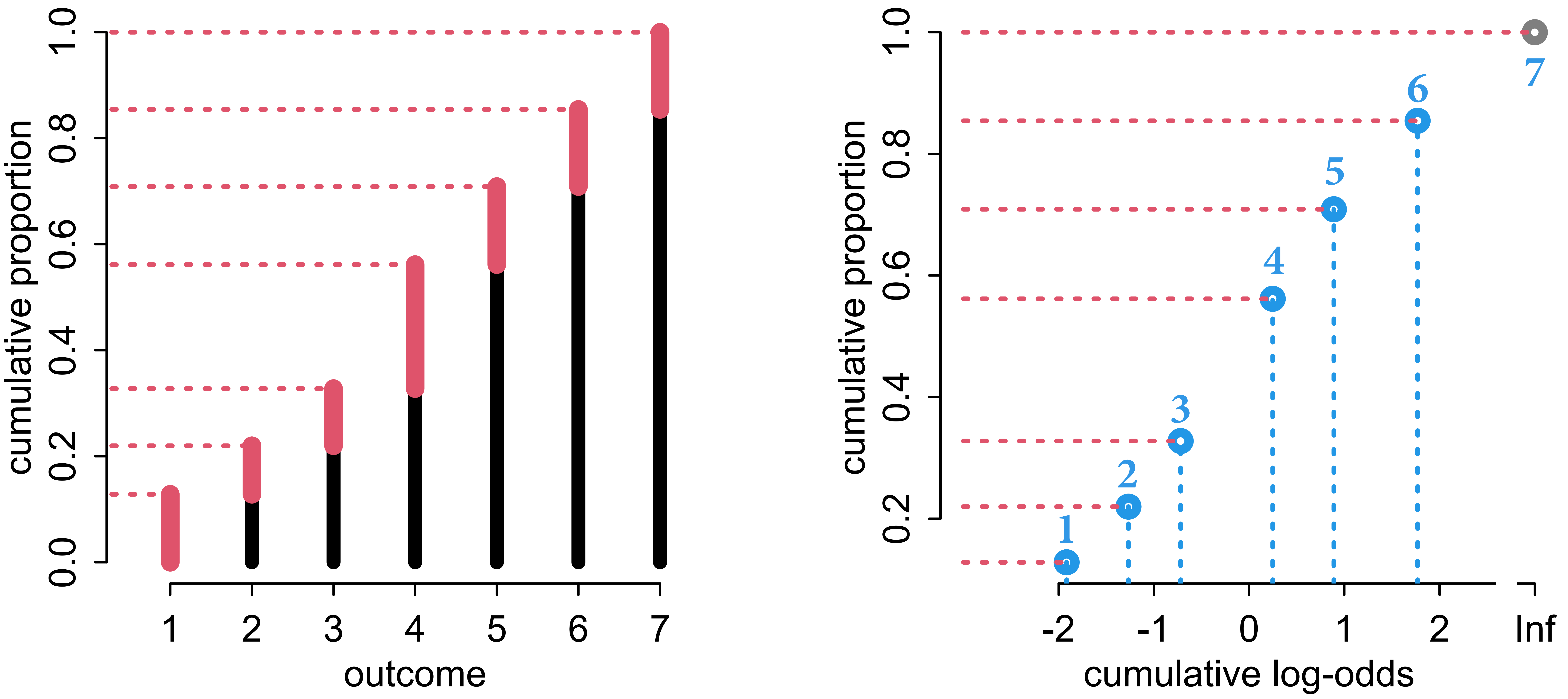

frequency
0
500
1500
2500
1
2
3
4
5
6
7
outcome



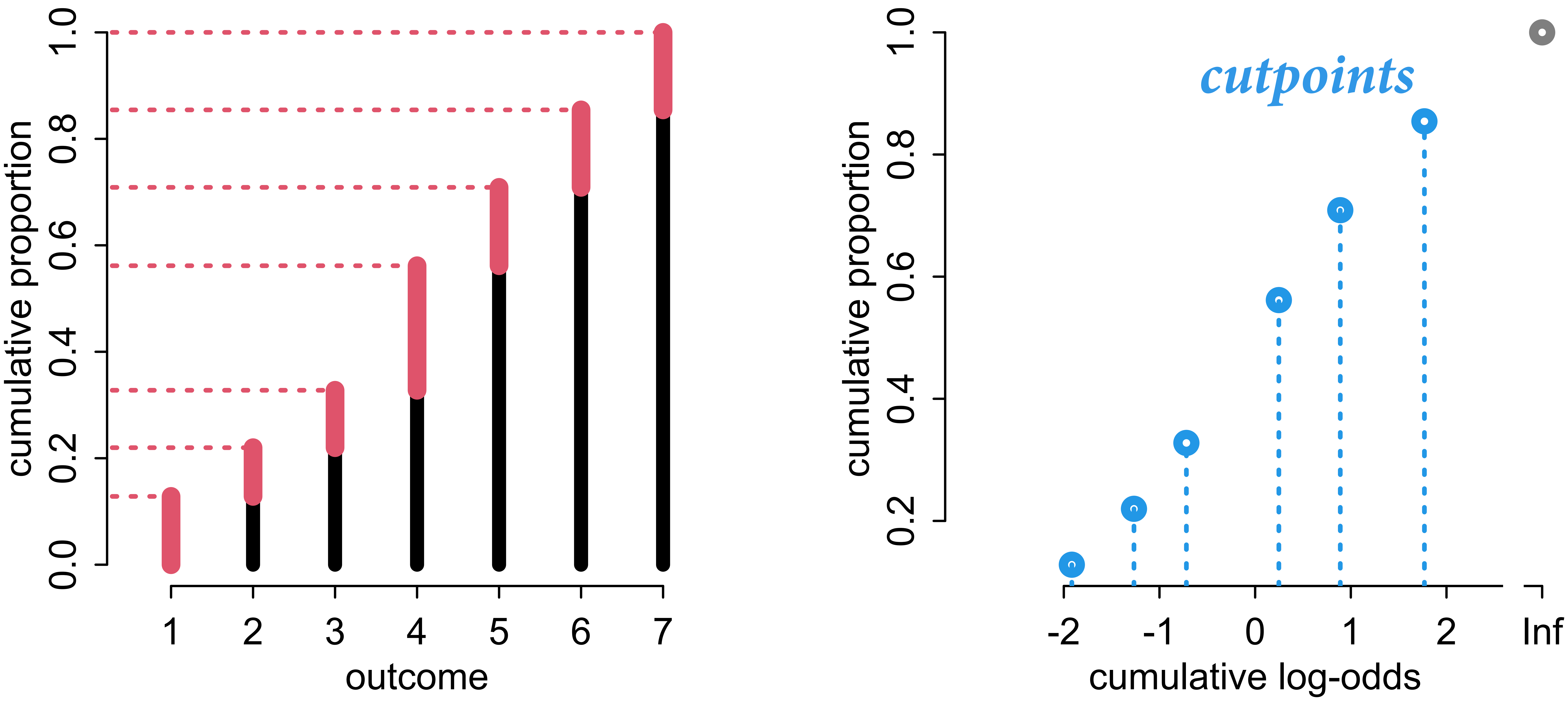
Ordered = Cumulative



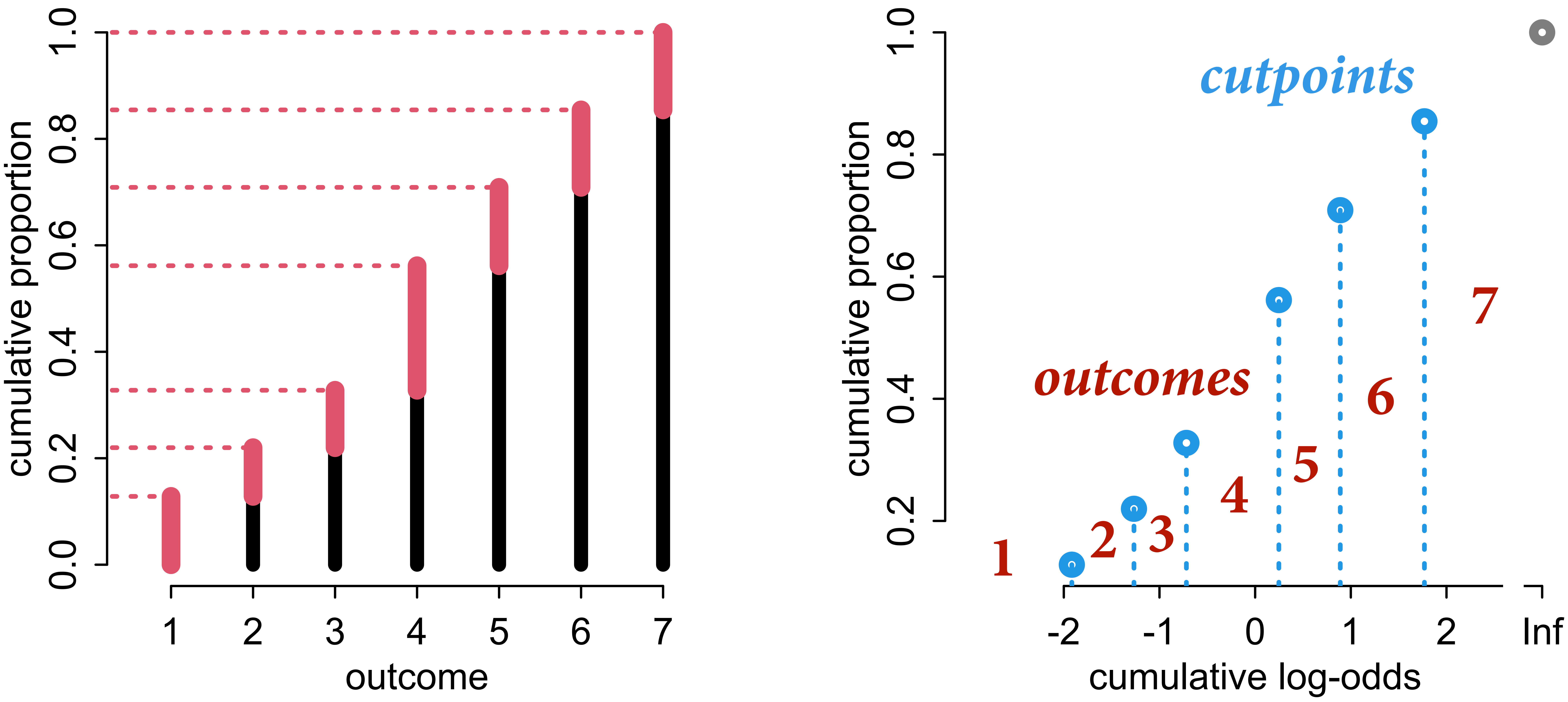




cumulative proportion
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
outcome
cumulative proportion
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
-2
-1
0
1
2
Inf
cumulative log-odds

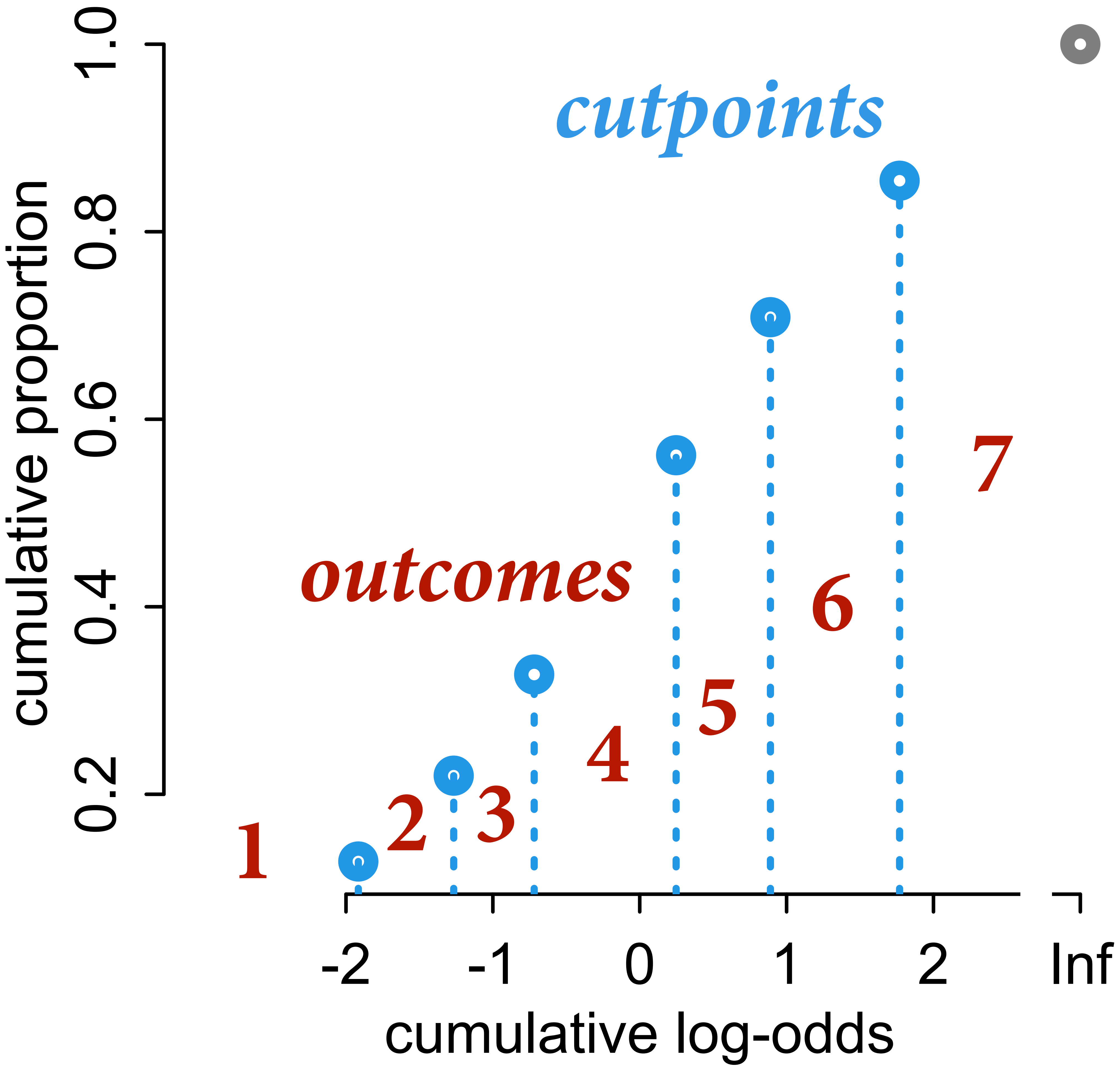


cumulative proportion
0.0
0.2
0.4
0.6
0.8
1.0
1 2 3 4 5 6 7
outcome
cutpoints
cumulative proportion
0.2
0.4
0.6
0.8
1.0
-2 -1 0 1 2 Inf
cumulative log-odds

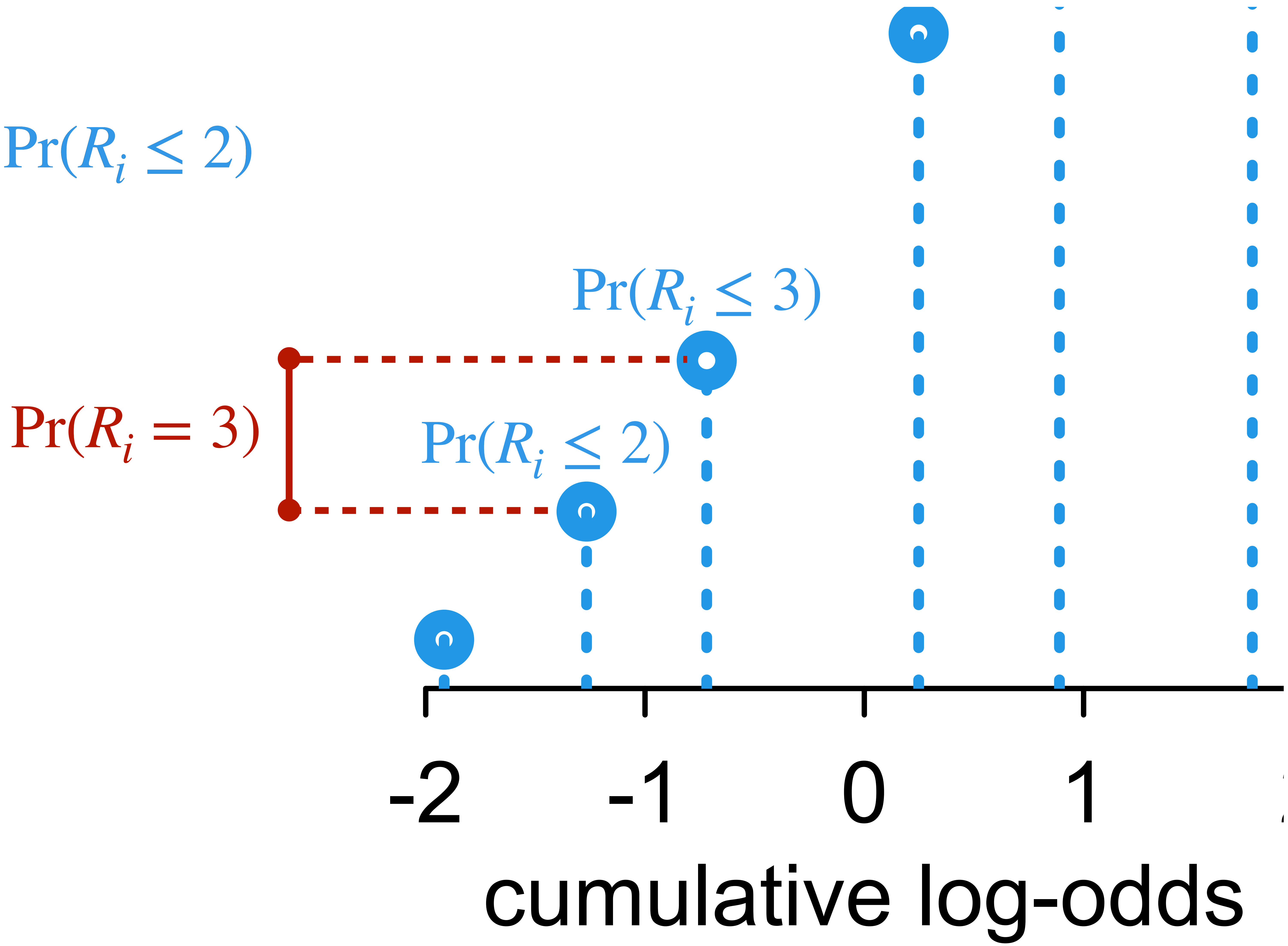


cumulative proportion
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
outcome
cutpoints
outcomes
cumulative proportion
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
-2
-1
0
1
2
Inf
cumulative log-odds

$$\Pr(R_i = k) = \Pr(R_i \leq k) - \Pr(R_i \leq k - 1)$$

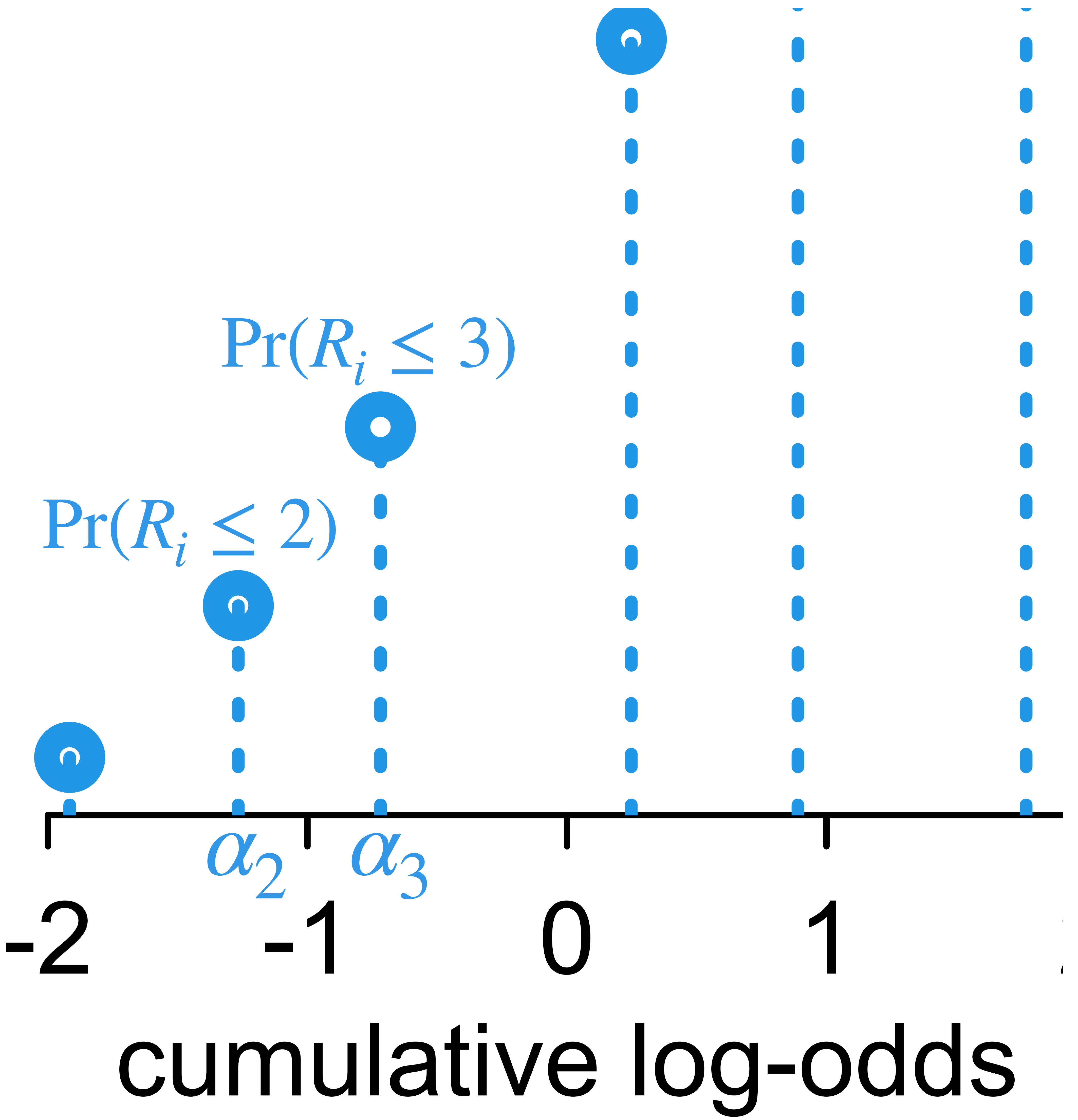


$$\Pr(R_i = 3) = \Pr(R_i \leq 3) - \Pr(R_i \leq 2)$$



$$\Pr(R_i = 3) = \Pr(R_i \leq 3) - \Pr(R_i \leq 2)$$

$$\underbrace{\log \frac{\Pr(R_i \leq k)}{1 - \Pr(R_i \leq k)}}_{\textit{cumulative log-odds}} = \underbrace{\alpha_k}_{\substack{\textit{cutpoint} \\ \textit{(to estimate)}}}$$



Where's the GLM?

So far just estimating the histogram

How to make it a function of variables?

(1) Stratify cutpoints

(2) Offset each cutpoint by value of linear model ϕ_i

Where's the GLM?

So far just estimating the histogram

$$\phi_i = \beta x_i$$

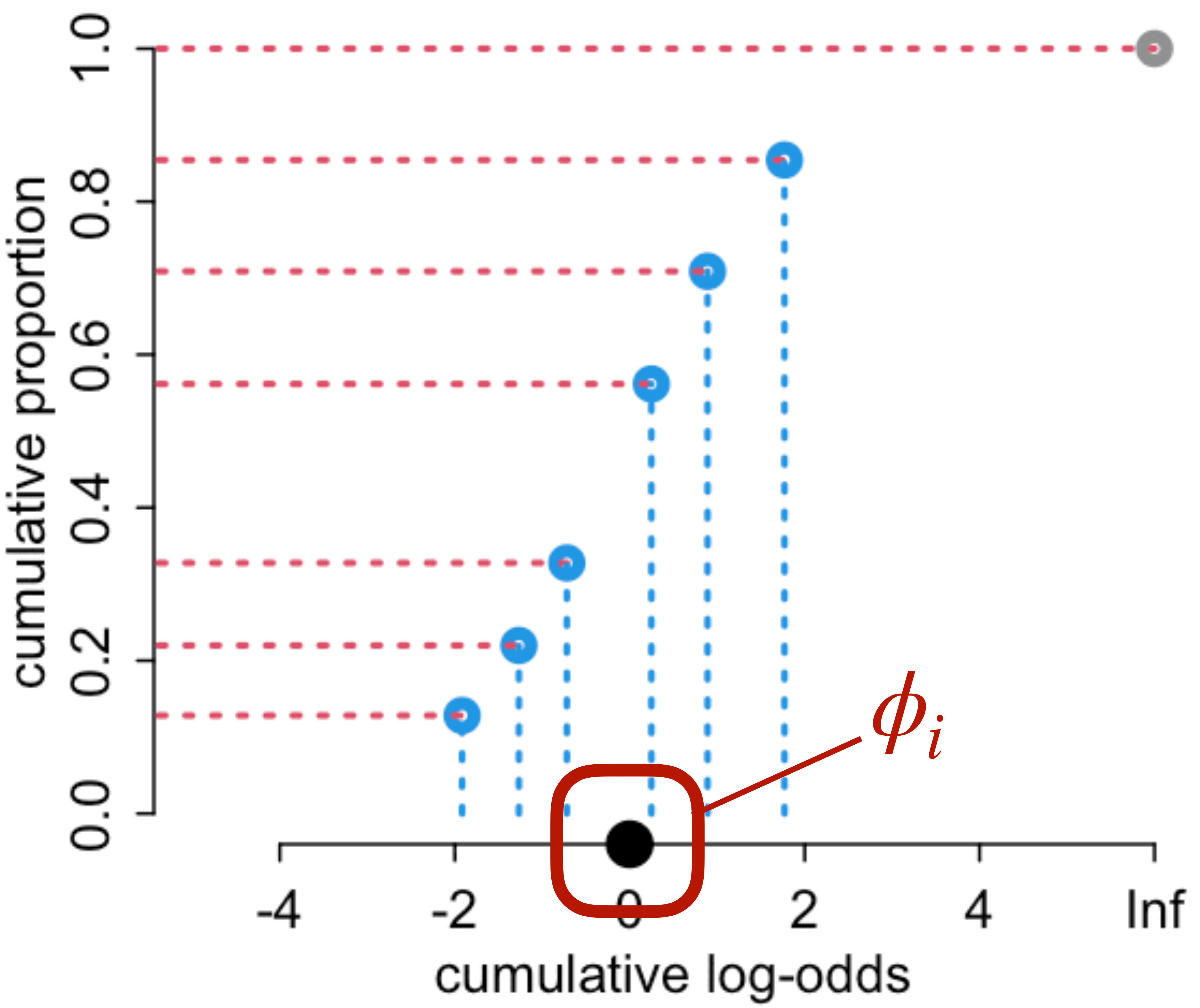
How to make it a function of variables?

$$\log \frac{\Pr(R_i \leq k)}{1 - \Pr(R_i \leq k)} = \alpha_k + \phi_i$$

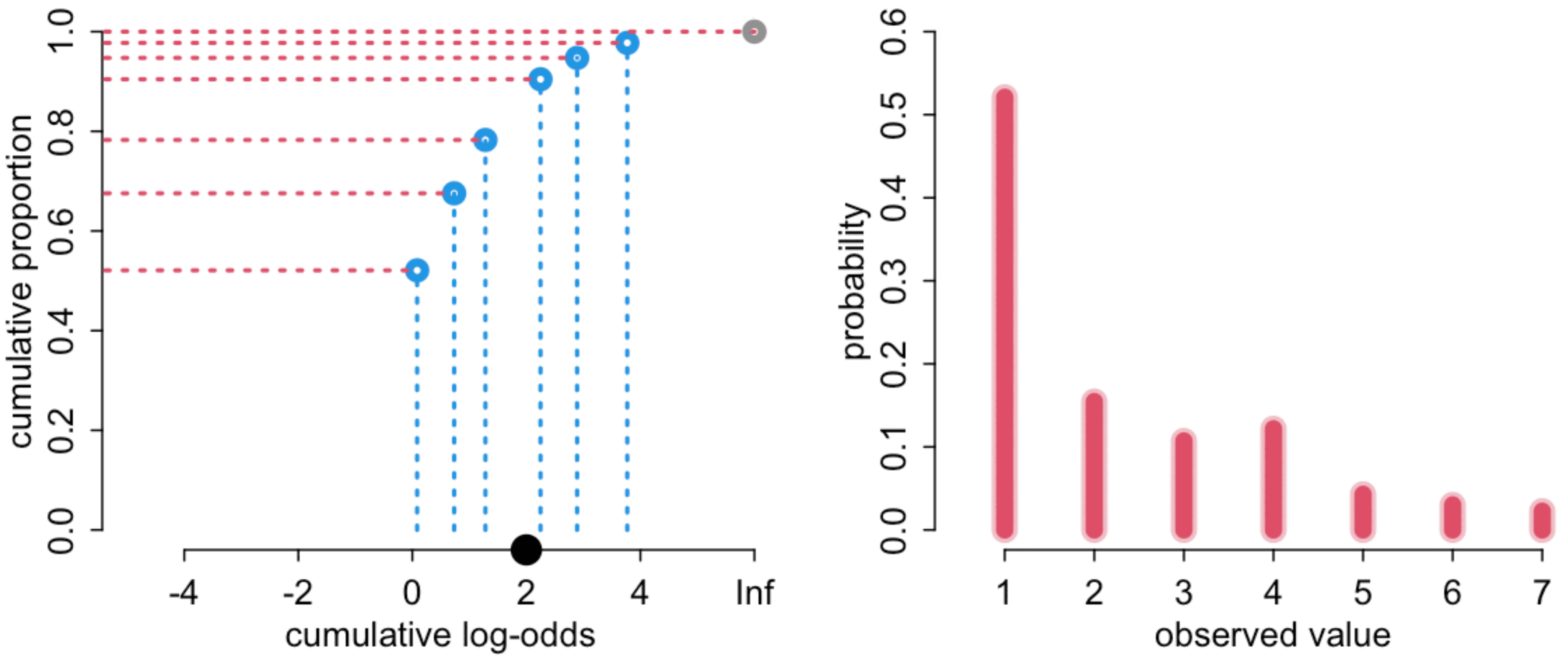
(1) Stratify cutpoints

(2) Offset each cutpoint by value of linear model ϕ_i

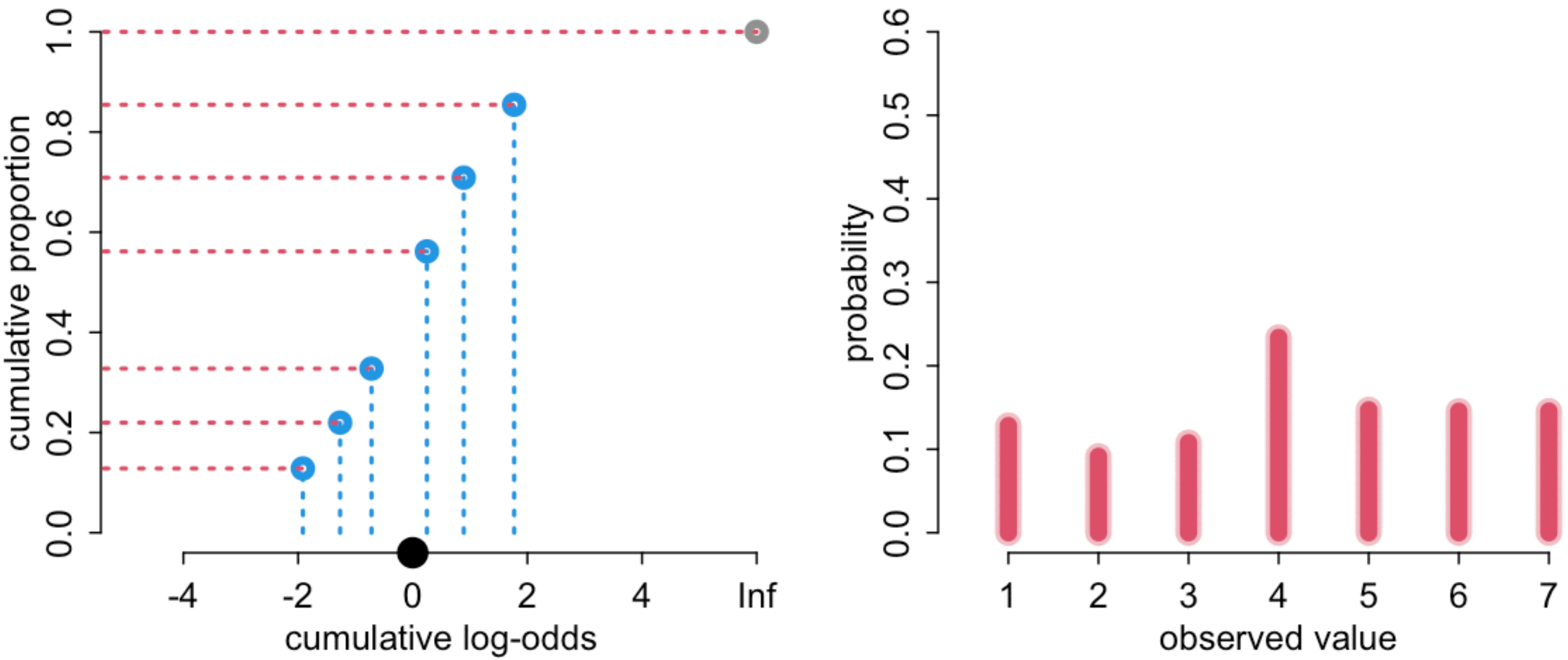
$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$



$$\log \frac{\Pr(R_i \le k)}{1 - \Pr(R_i \le k)} = \alpha_k + \phi_i$$

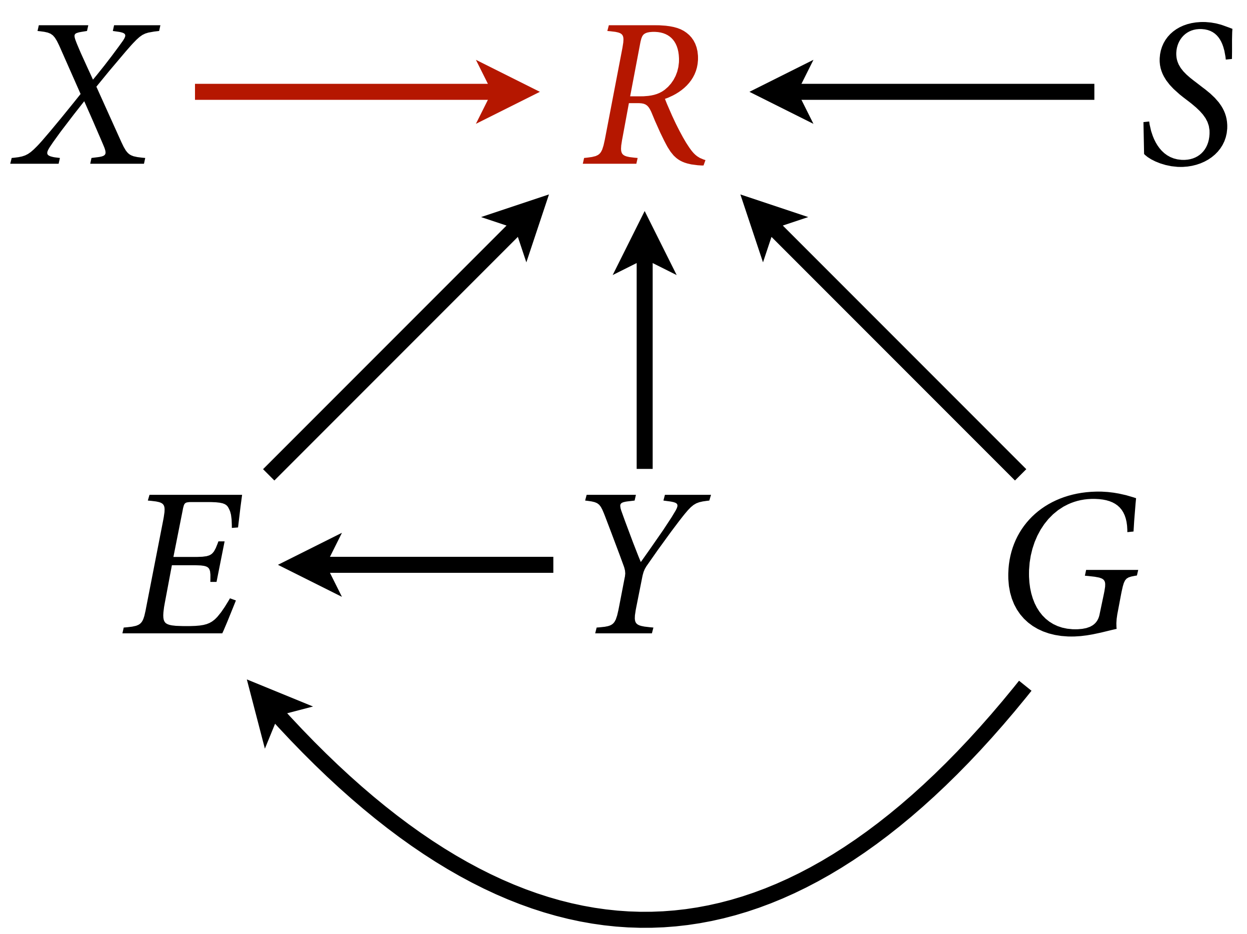


cumulative proportion
1.0
0.8
0.6
0.4
0.2
0.0
-4
-2
0
2
4
Inf
cumulative log-odds
probability
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
5
6
7
observed value



cumulative proportion
0.0
0.2
0.4
0.6
0.8
1.0
-4
-2
0
2
4
Inf
cumulative log-odds
probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
1
2
3
4
5
6
7
observed value

Start off easy:



$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_A A_i + \beta_C C_i + \beta_I I_i$$

$$\beta_{\_} \sim \text{Normal}(0{,}0.5)$$

$$\alpha_j \sim \text{Normal}(0{,}1)$$

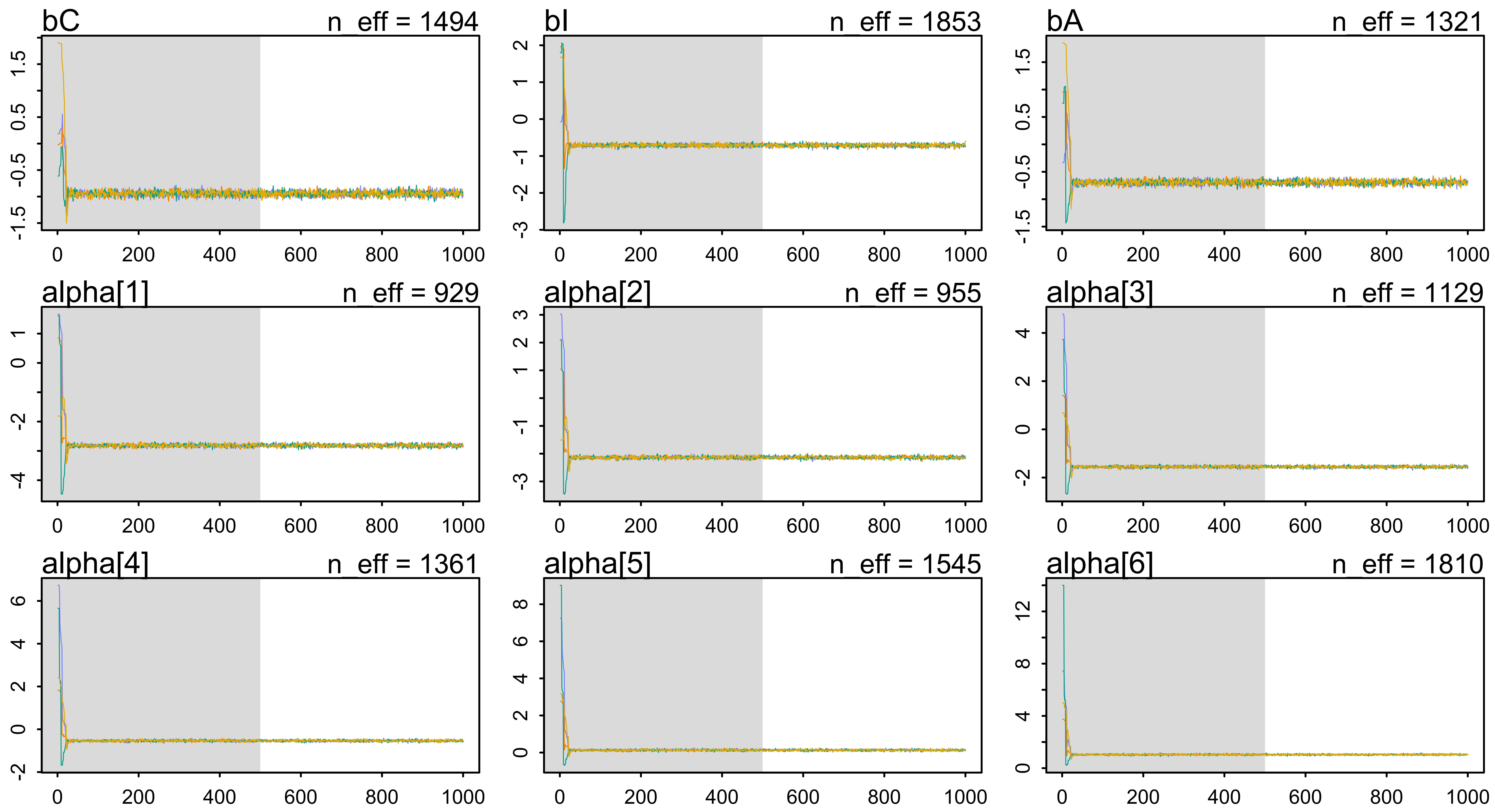
```r
data(Trolley)
d <- Trolley
dat <- list(
    R = d$response,
    A = d$action,
    I = d$intention,
    C = d$contact
)

mRX <- ulam(
    alist(

        R ~ dordlogit(phi,alpha),
        phi <- bA*A + bI*I + bC*C,
        c(bA,bI,bC) ~ normal(0,0.5),
        alpha ~ normal(0,1)

    ) , data=dat , chains=4 , cores=4 )
```

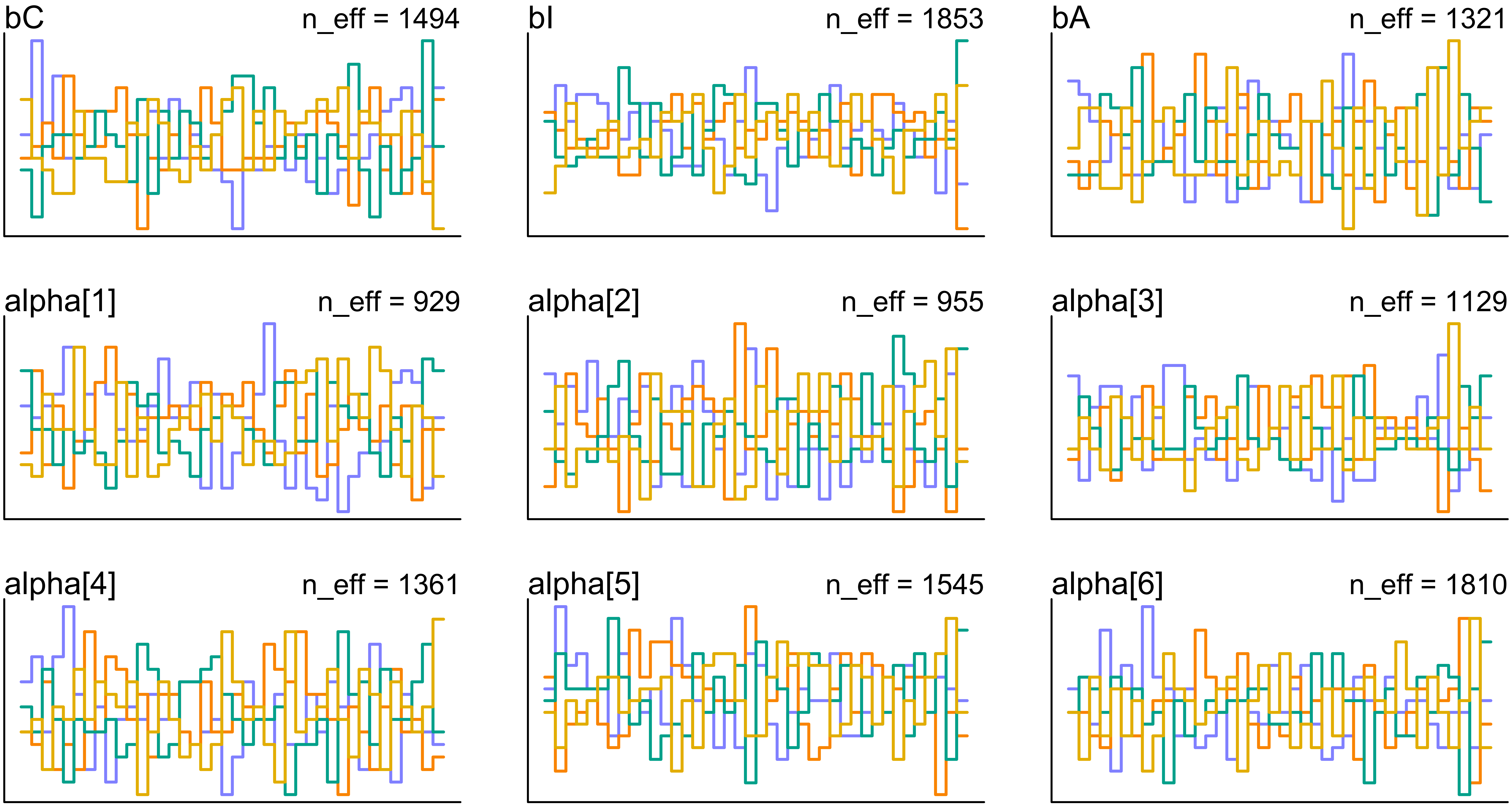
$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_A A_i + \beta_C C_i + \beta_I I_i$$

$$\beta\_ \sim \text{Normal}(0{,}0.5)$$

$$\alpha_j \sim \text{Normal}(0{,}1)$$


bC
n_eff = 1494
bI
n_eff = 1853
bA
n_eff = 1321
alpha[1]
n_eff = 929
alpha[2]
n_eff = 955
alpha[3]
n_eff = 1129
alpha[4]
n_eff = 1361
alpha[5]
n_eff = 1545
alpha[6]
n_eff = 1810



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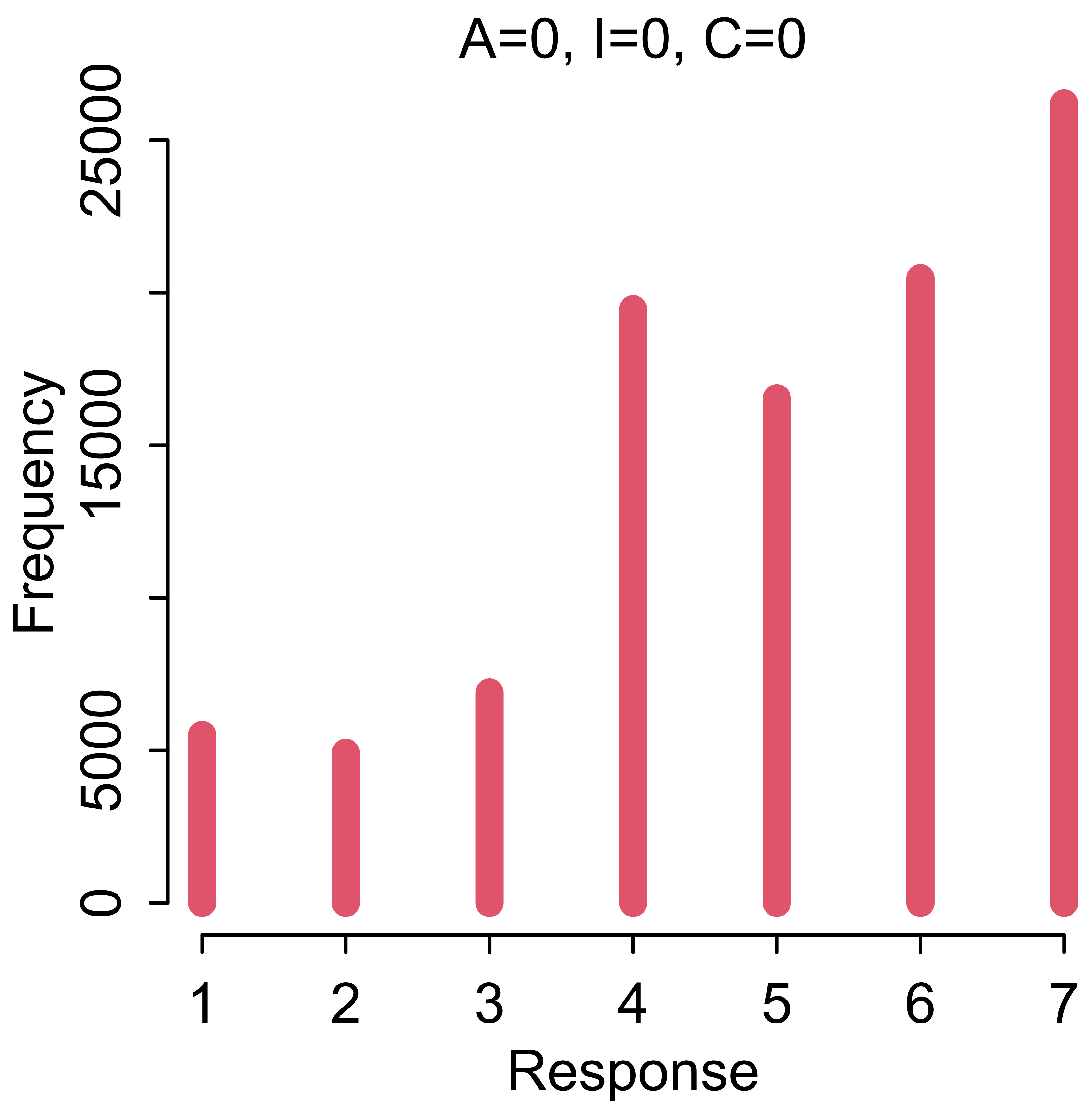
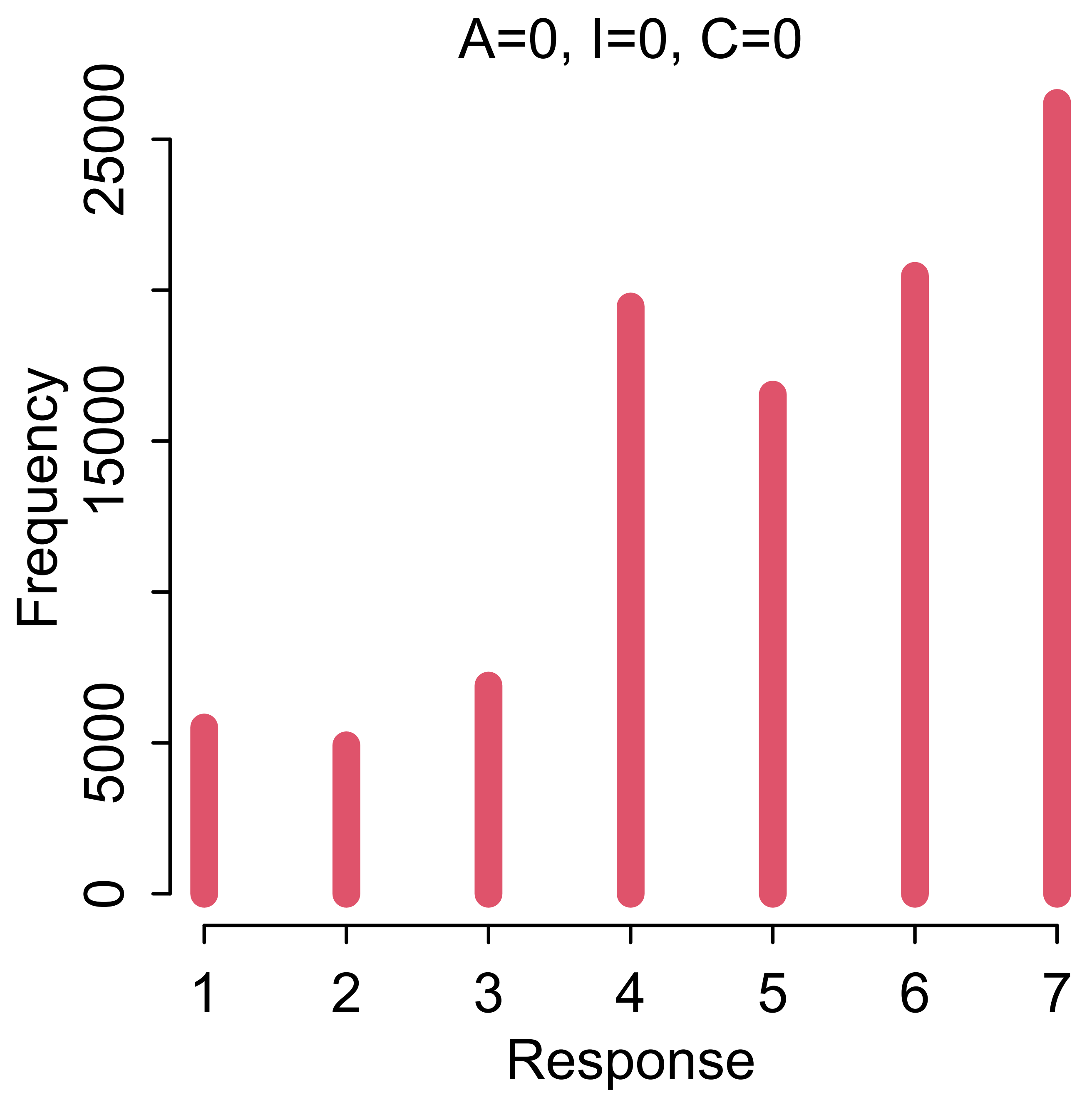
n_eff = 955
alpha[3]
n_eff = 1129
alpha[4]
n_eff = 1361
alpha[5]
n_eff = 1545
alpha[6]
n_eff = 1810

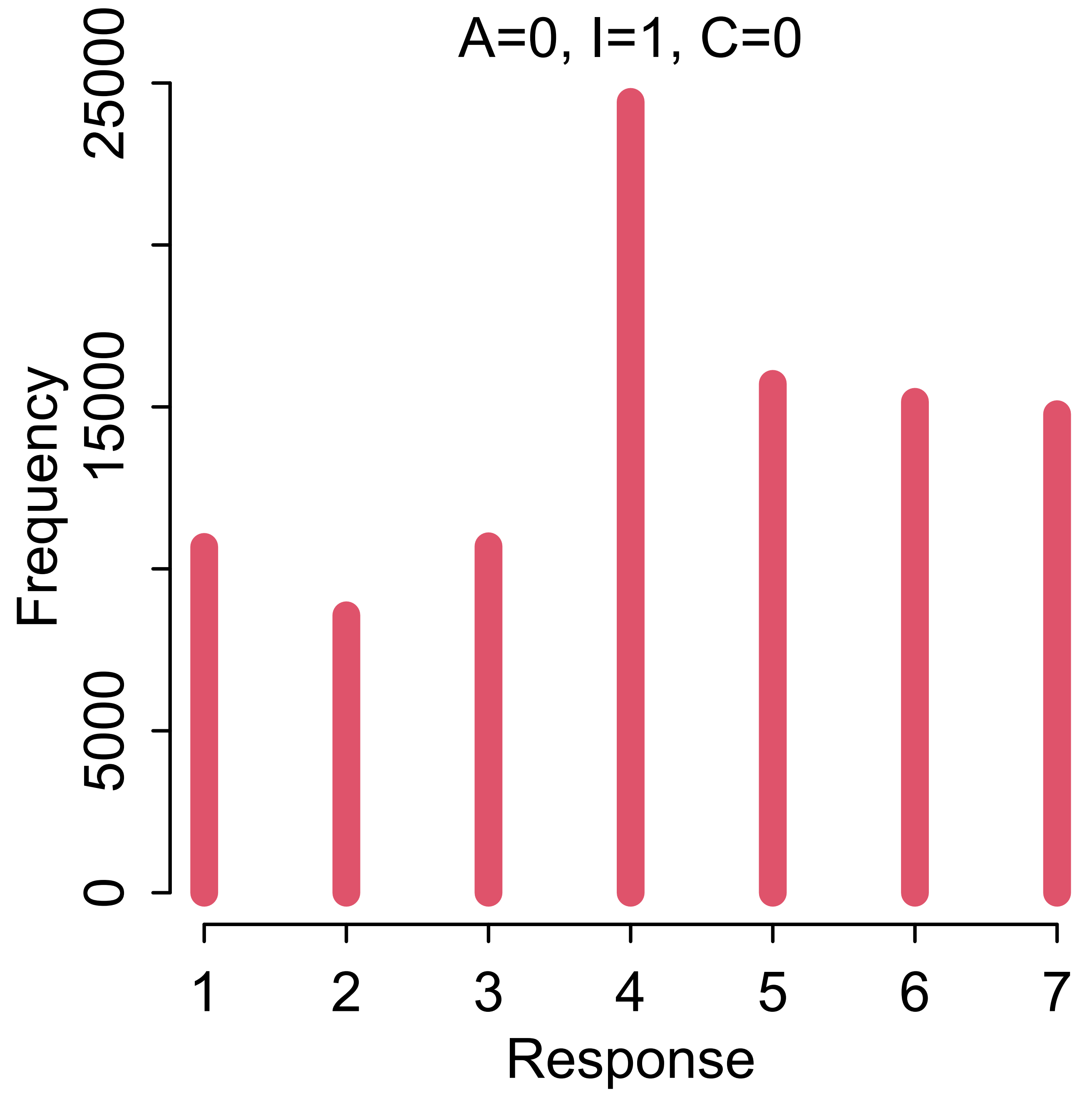
```r
data(Trolley)
d <- Trolley
dat <- list(
    R = d$response,
    A = d$action,
    I = d$intention,
    C = d$contact
)

mRX <- ulam(
    alist(

        R ~ dordlogit(phi,alpha),
        phi <- bA*A + bI*I + bC*C,
        c(bA,bI,bC) ~ normal(0,0.5),
        alpha ~ normal(0,1)

    ) , data=dat , chains=4 , cores=4 )
```

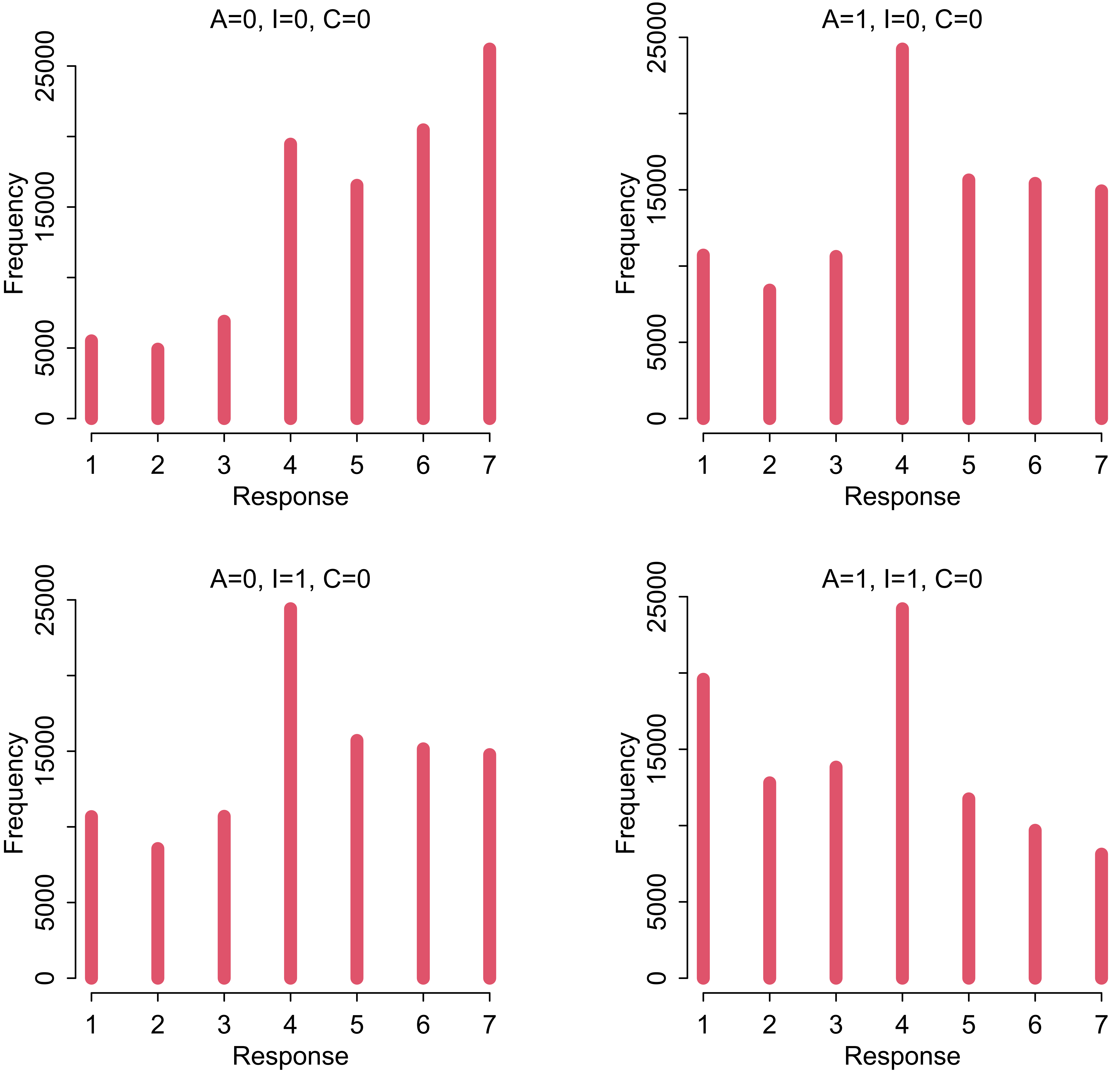
```
> precis(mRX,2)
           mean   sd  5.5% 94.5% n_eff Rhat4
bC        -0.94 0.05 -1.02 -0.87  1494     1
bI        -0.71 0.04 -0.77 -0.65  1853     1
bA        -0.69 0.04 -0.76 -0.63  1321     1
alpha[1]  -2.82 0.05 -2.89 -2.74   929     1
alpha[2]  -2.14 0.04 -2.20 -2.07   955     1
alpha[3]  -1.56 0.04 -1.62 -1.49  1129     1
alpha[4]  -0.54 0.04 -0.59 -0.48  1361     1
alpha[5]   0.13 0.04  0.07  0.19  1545     1
alpha[6]   1.04 0.04  0.97  1.10  1810     1
```

```
# plot predictive distributions for each treatment

vals <- c(0,0,0)
Rsim <- mcreplicate( 100 ,
sim(mRX,data=list(A=vals[1],I=vals[2],C=vals[3])) ,
mc.cores=6 )

simplehist(as.vector(Rsim),lwd=8,col=2,xlab="Response")
mtext(concat("A=",vals[1],", I=",vals[2],",
C=",vals[3]))
```

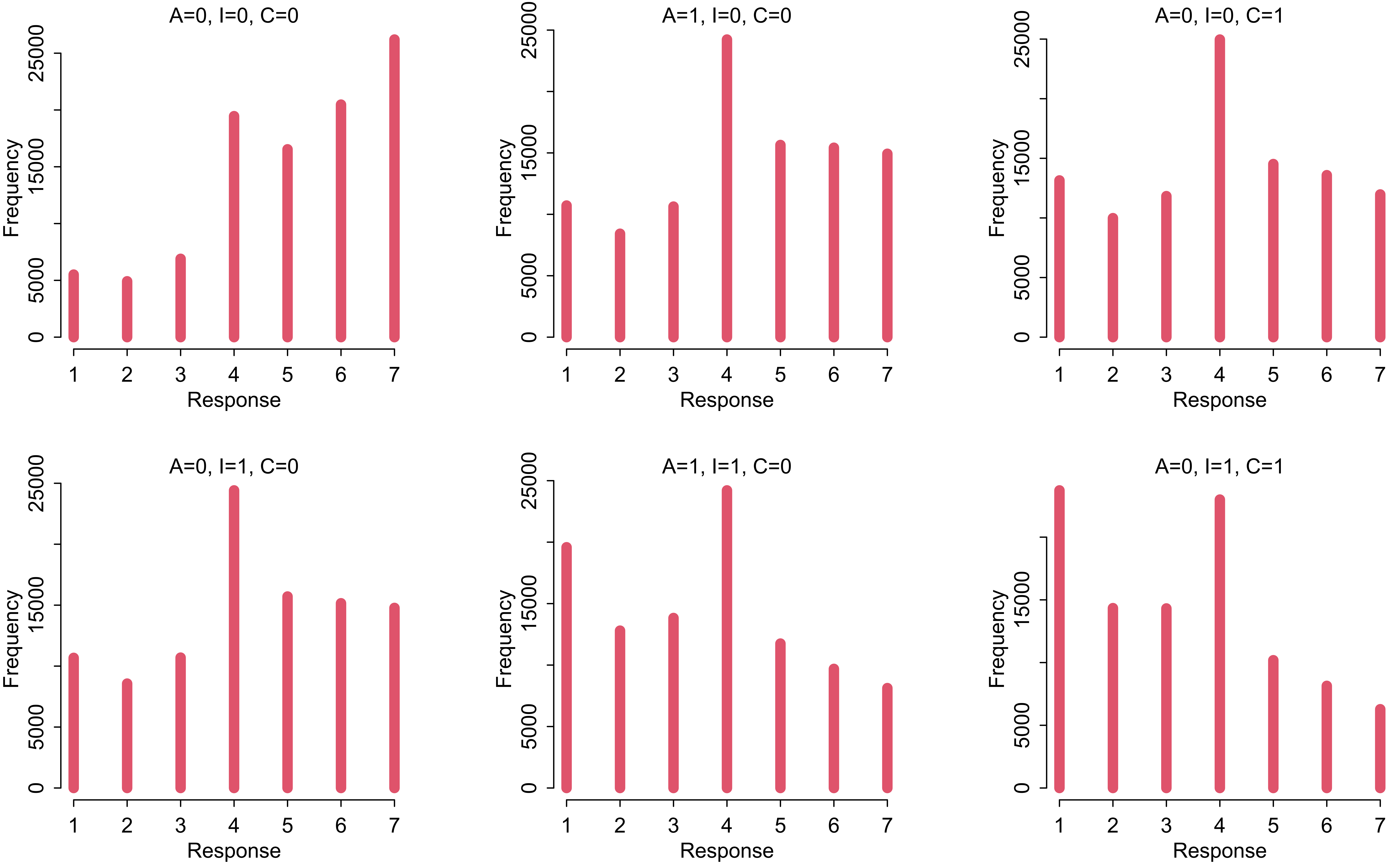



A=0, I=0, C=0
Frequency
0
5000
15000
25000
1
2
3
4
5
6
7
Response

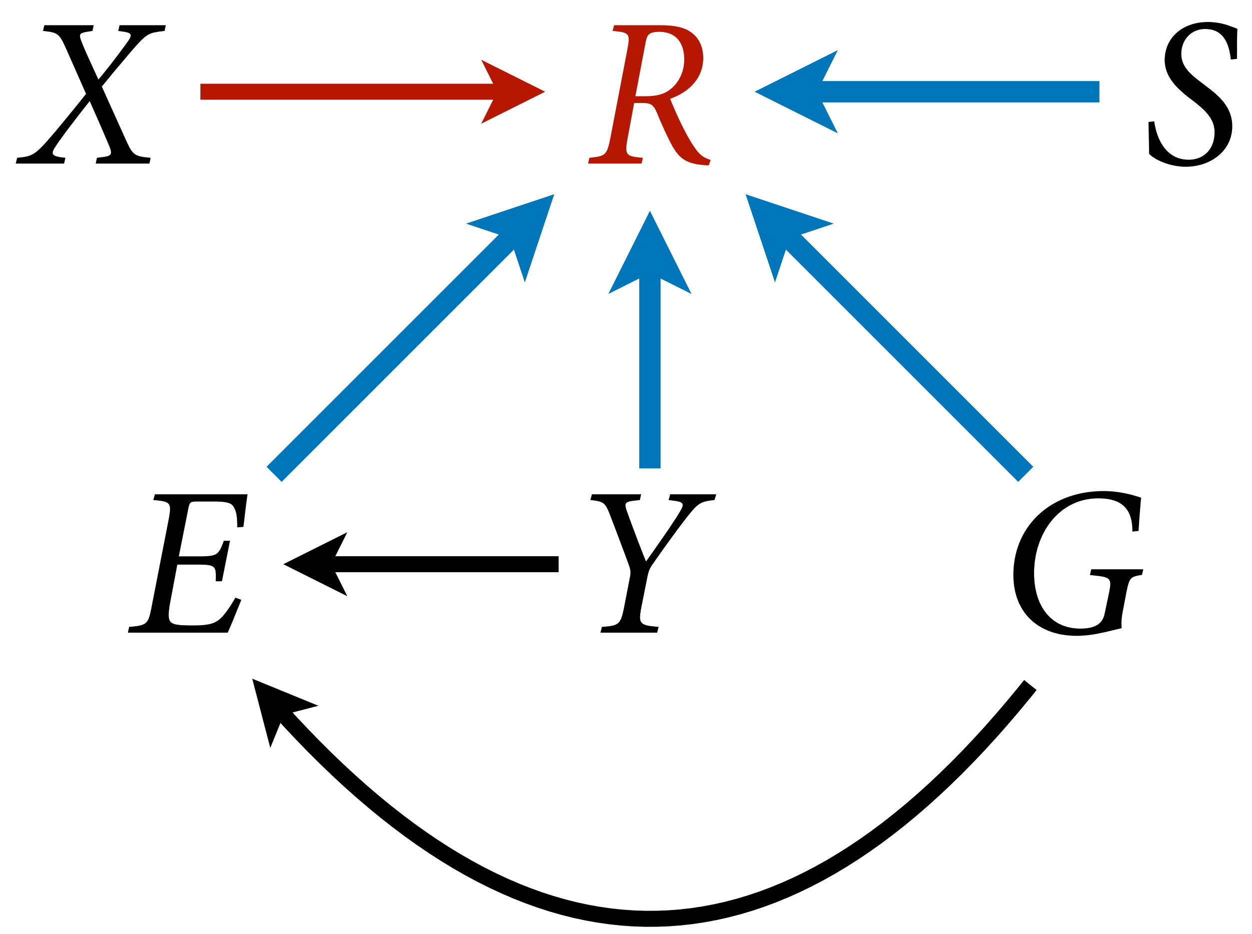

A=0, I=1, C=0
Frequency
0
5000
15000
25000
1
2
3
4
5
6
7
Response



A=0, I=0, C=0
A=1, I=0, C=0
A=0, I=1, C=0
A=1, I=1, C=0
Frequency
Response
0
5000
15000
25000
1
2
3
4
5
6
7


A=0, I=0, C=0
A=1, I=0, C=0
A=0, I=0, C=1
A=0, I=1, C=0
A=1, I=1, C=0
A=0, I=1, C=1
Frequency
Response
0
5000
15000
25000
1
2
3
4
5
6
7

What about the competing causes?



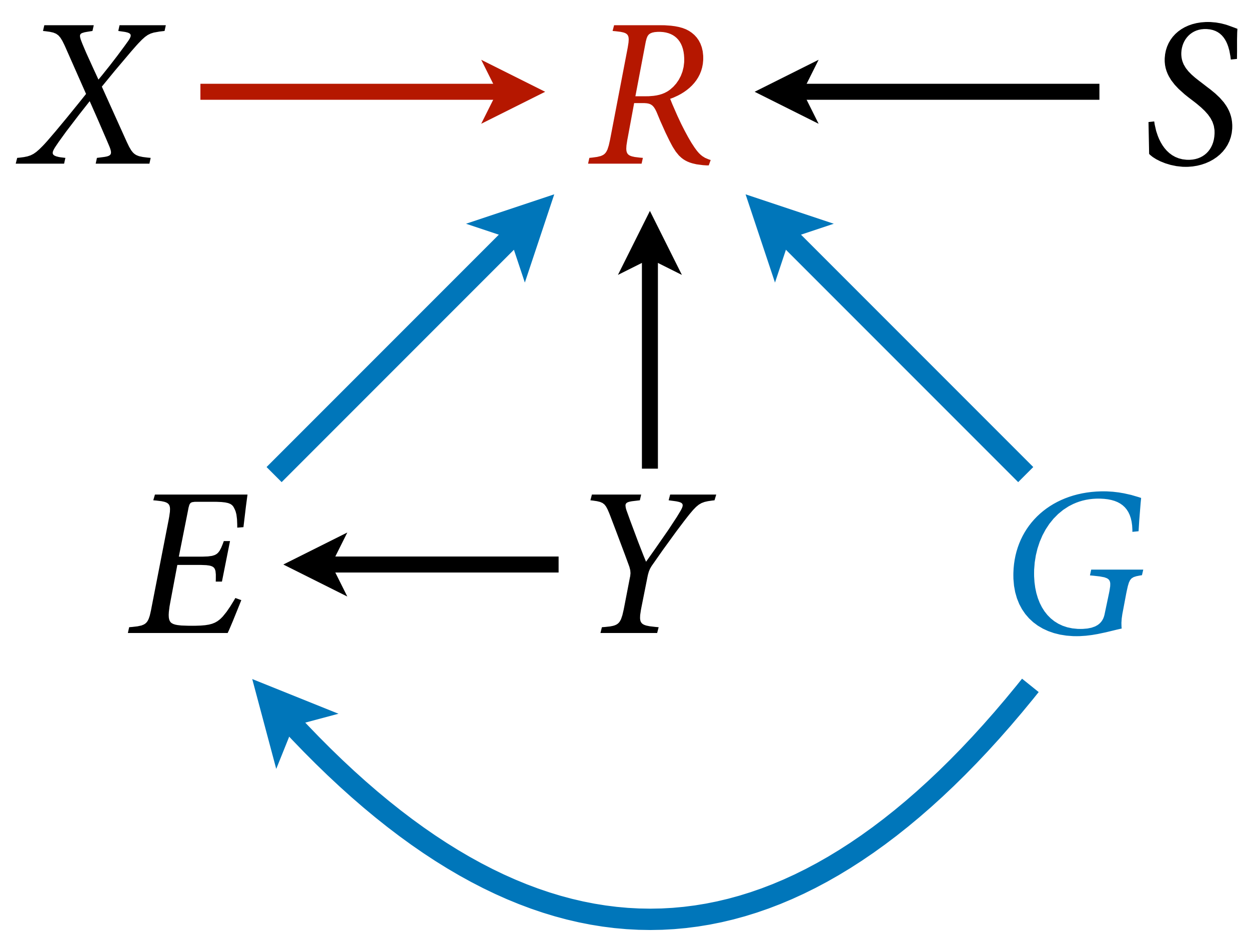
$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_A A_i + \beta_C C_i + \beta_I I_i$$

$$\beta \sim \text{Normal}(0,0.5)$$

$$\alpha_j \sim \text{Normal}(0,1)$$

Total effect of gender:



$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_{A,G[i]}A_i + \beta_{C,G[i]}C_i + \beta_{I,G[i]}I_i$$

$$\beta_{\_} \sim \text{Normal}(0,0.5)$$

$$\alpha_j \sim \text{Normal}(0,1)$$

```r
# total effect of gender
dat$G <- iflelse(d$male==1,2,1)
mRXG <- ulam(
    alist(

        R ~ dordlogit(phi,alpha),
        phi <- bA[G]*A + bI[G]*I + bC[G]*C,
        bA[G] ~ normal(0,0.5),
        bI[G] ~ normal(0,0.5),
        bC[G] ~ normal(0,0.5),
        alpha ~ normal(0,1)

    ) , data=dat , chains=4 , cores=4 )
```

$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_{A,G[i]} A_i + \beta_{C,G[i]} C_i + \beta_{I,G[i]} I_i$$

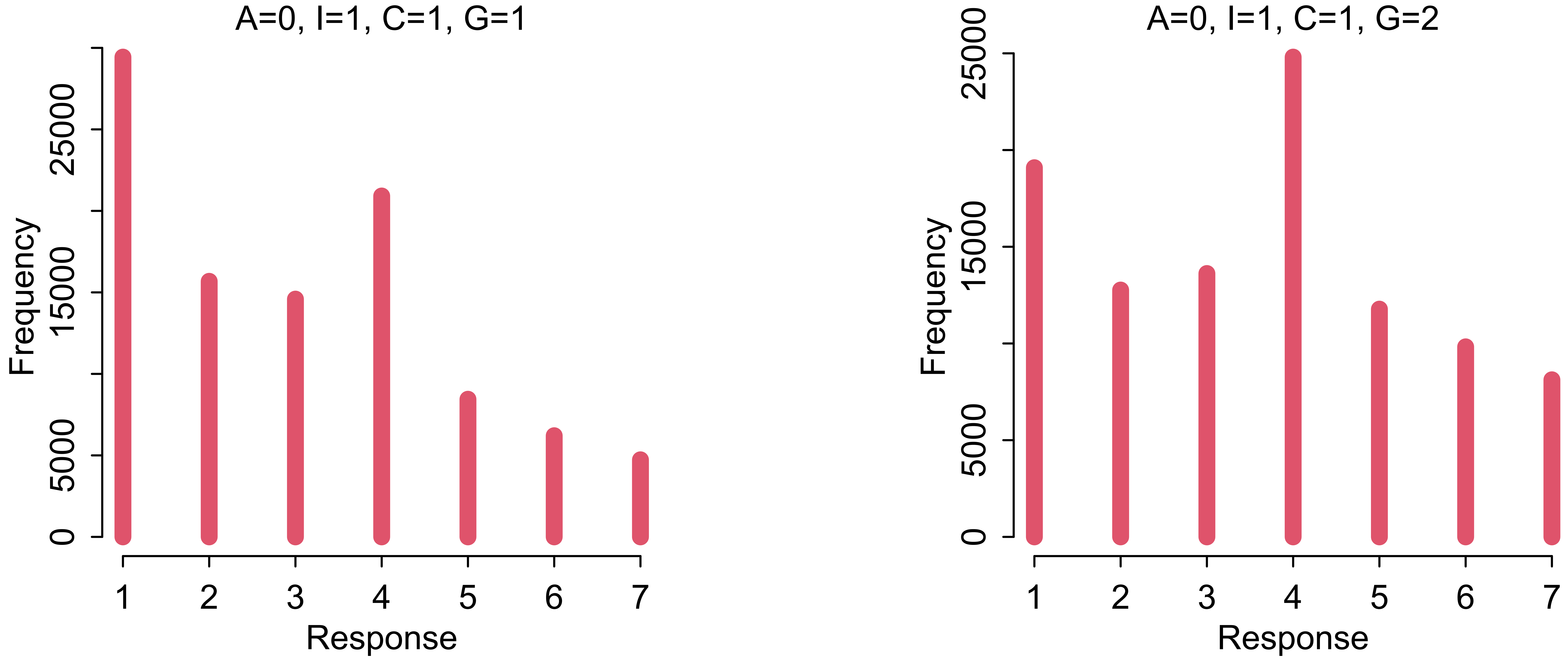
$$\beta_{\_} \sim \text{Normal}(0,0.5)$$

$$\alpha_j \sim \text{Normal}(0,1)$$

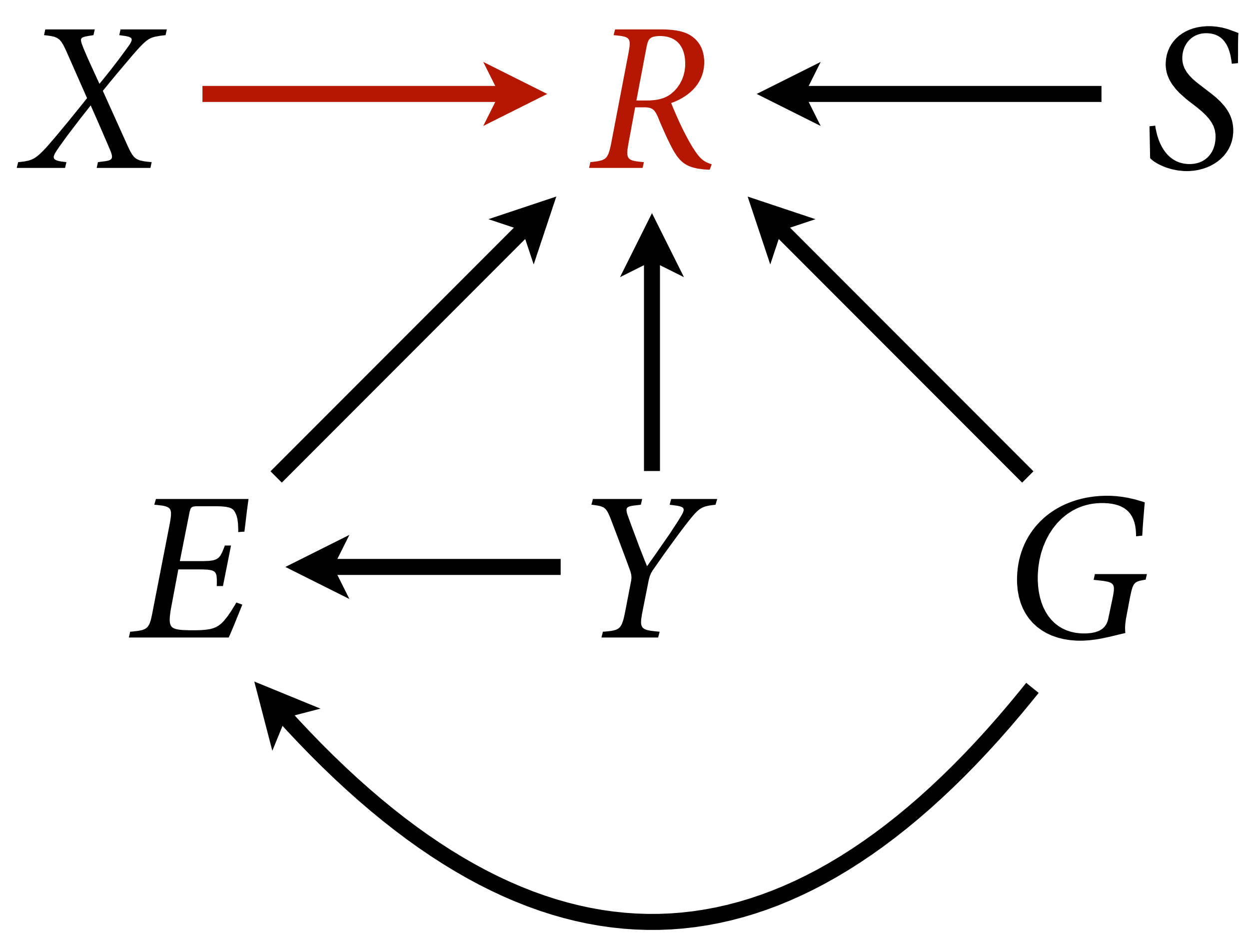
```r
# total effect of gender
dat$G <- ifelse(d$male==1,2,1)
mRXG <- ulam(
    alist(

        R ~ dordlogit(phi,alpha),
        phi <- bA[G]*A + bI[G]*I + bC[G]*C,
        bA[G] ~ normal(0,0.5),
        bI[G] ~ normal(0,0.5),
        bC[G] ~ normal(0,0.5),
        alpha ~ normal(0,1)

    ) , data=dat , chains=4 , cores=4 )
```

```
> precis(mRXG,2)
          mean   sd  5.5% 94.5% n_eff Rhat4
bA[1]    -0.88 0.05 -0.96 -0.80  1858  1.00
bA[2]    -0.53 0.05 -0.61 -0.45  1724  1.00
bI[1]    -0.90 0.05 -0.97 -0.82  2189  1.00
bI[2]    -0.55 0.05 -0.63 -0.48  2382  1.00
bC[1]    -1.06 0.07 -1.17 -0.95  2298  1.00
bC[2]    -0.84 0.06 -0.94 -0.74  2000  1.00
alpha[1] -2.83 0.05 -2.90 -2.75  1054  1.01
alpha[2] -2.15 0.04 -2.21 -2.08  1104  1.00
alpha[3] -1.56 0.04 -1.62 -1.50  1076  1.00
alpha[4] -0.53 0.04 -0.59 -0.47  1080  1.00
alpha[5]  0.14 0.04  0.09  0.20  1216  1.00
alpha[6]  1.06 0.04  1.00  1.12  1532  1.00
```

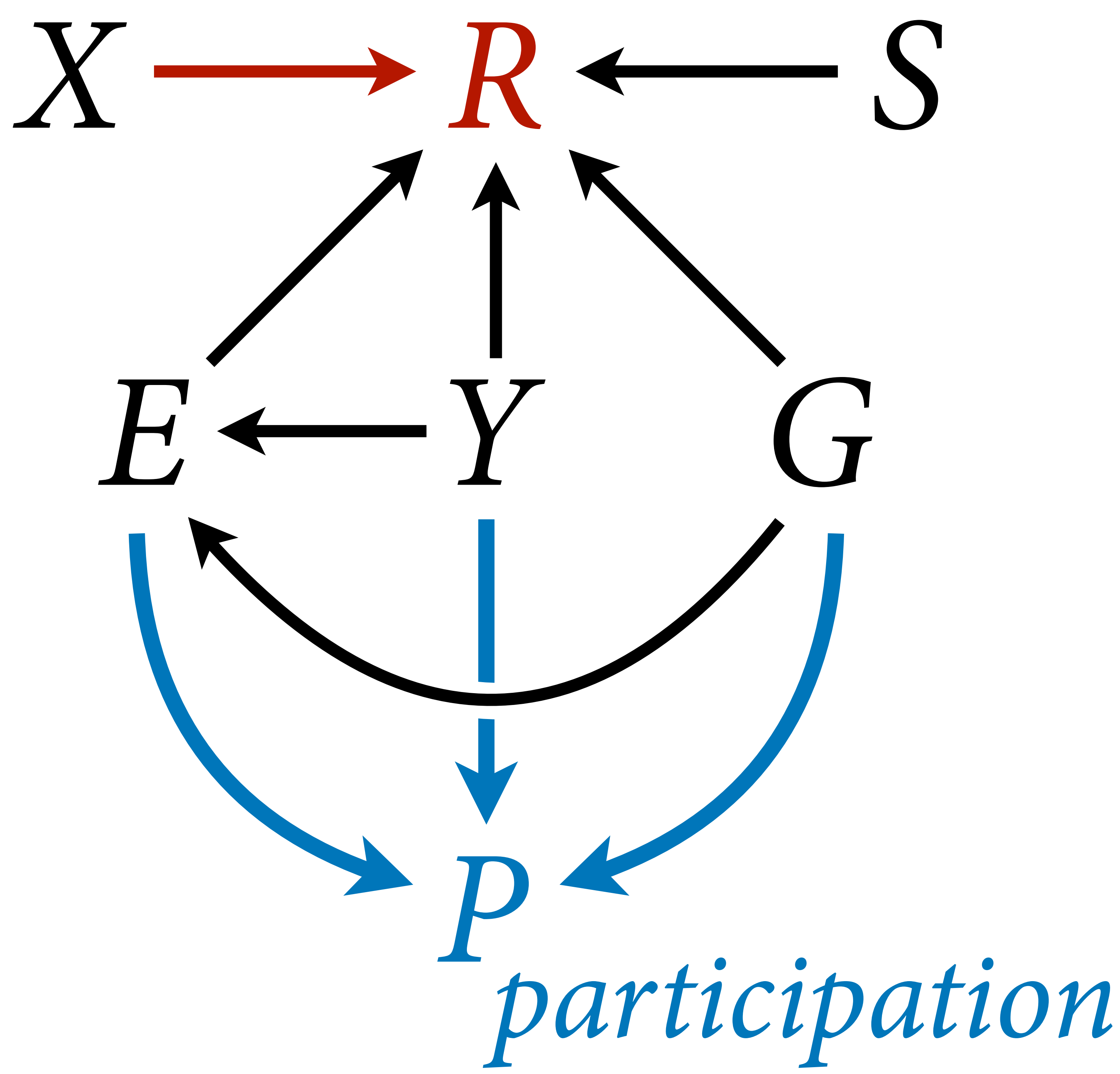
$[i],i$


A=0, I=1, C=1, G=1
Frequency
0
5000
15000
25000
1
2
3
4
5
6
7
Response
A=0, I=1, C=1, G=2
Frequency
0
5000
15000
25000
1
2
3
4
5
6
7
Response

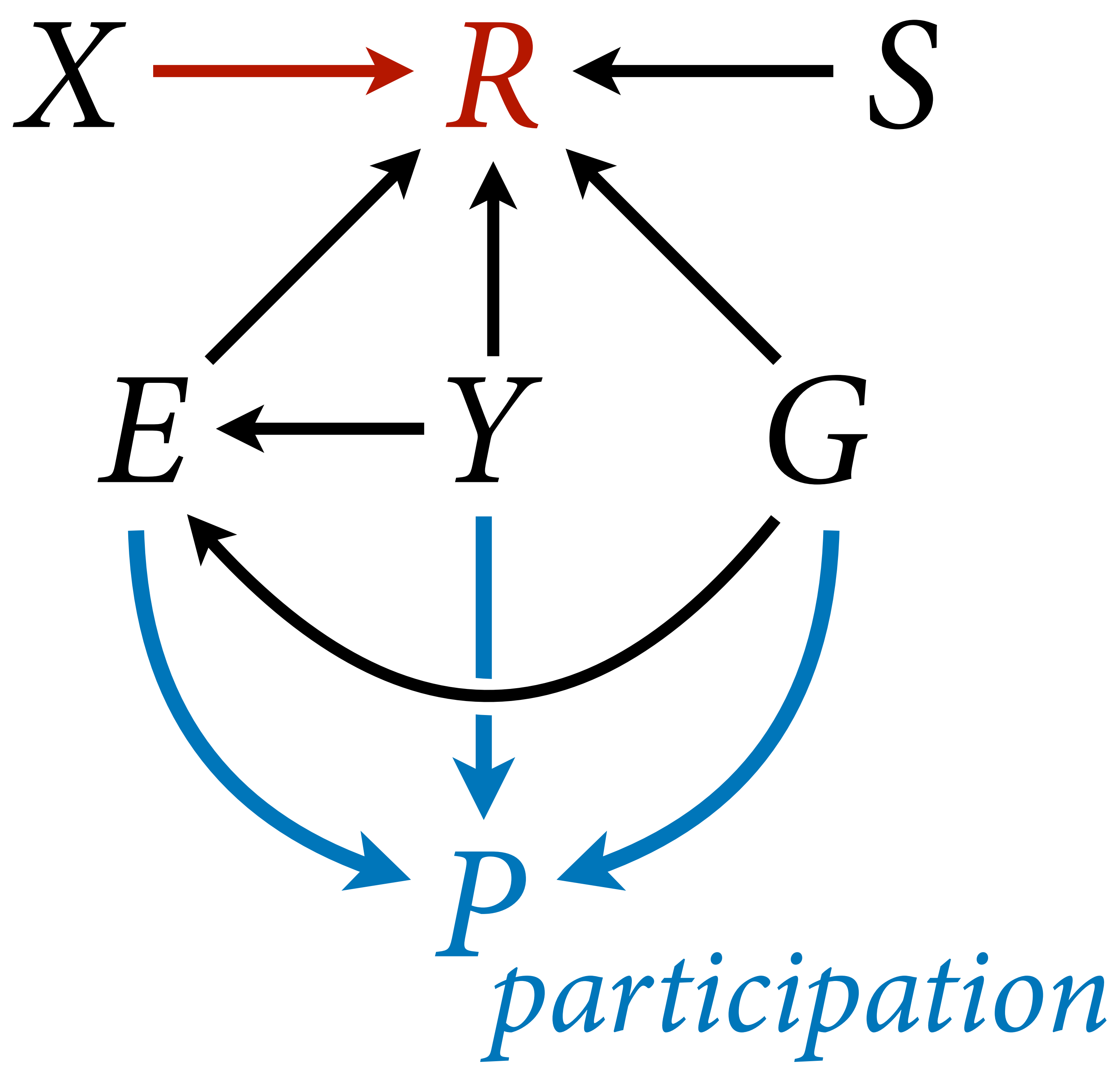
Hang on! This is a voluntary sample



Hang on! This is a voluntary sample



Hang on! This is a voluntary sample



Conditioning on P makes E,Y,G covary in sample

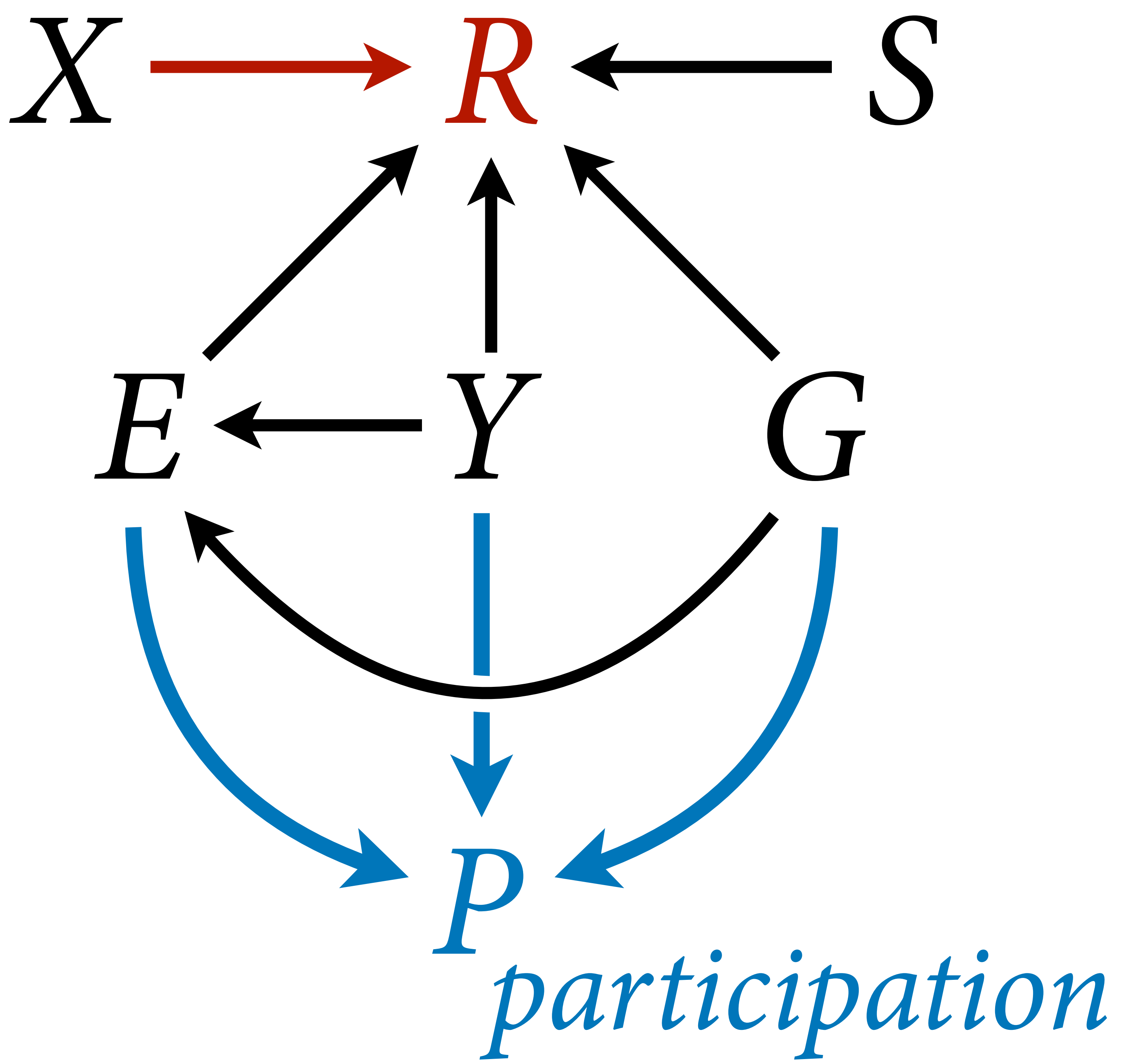
Endogenous selection

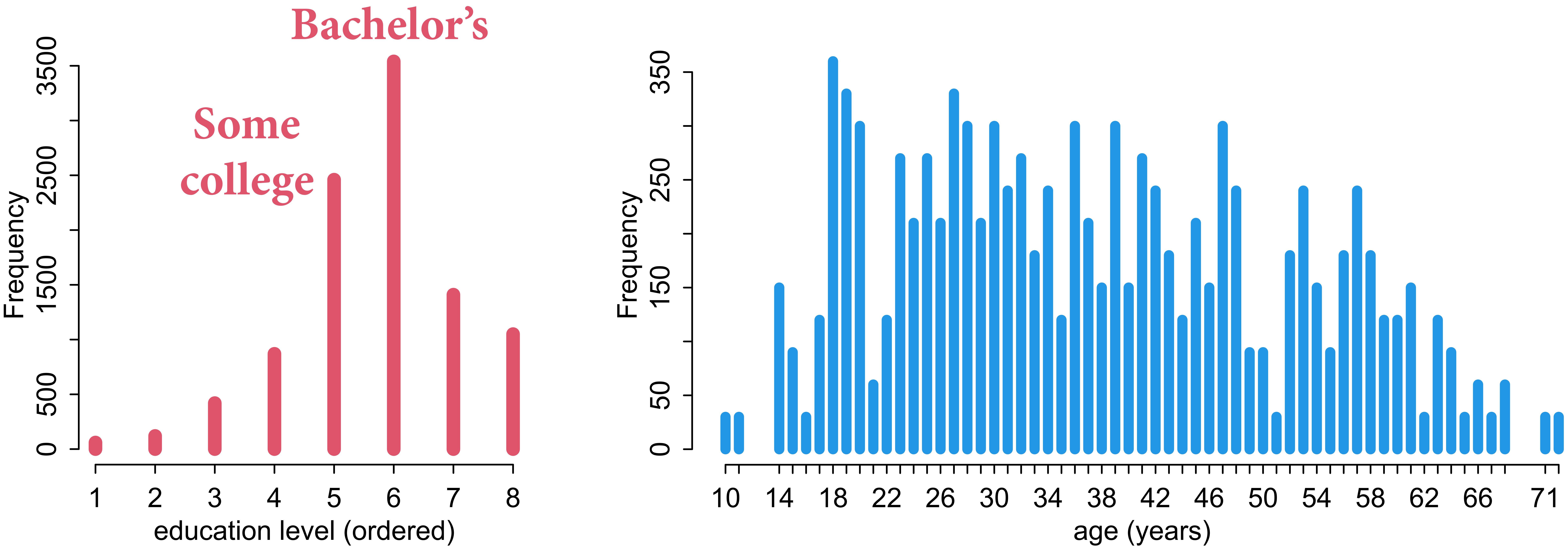
Sample is selected on a collider

Induces misleading associations among variables

Not possible here to estimate total effect of *G*, BUT can get direct effect

Need to stratify by *E* and *Y* and *G*




Bachelor's
Some college
Frequency
0
500
1500
2500
3500
1
2
3
4
5
6
7
8
education level (ordered)
Frequency
0
50
150
250
350
10
14
18
22
26
30
34
38
42
46
50
54
58
62
66
71
age (years)

PAUSE

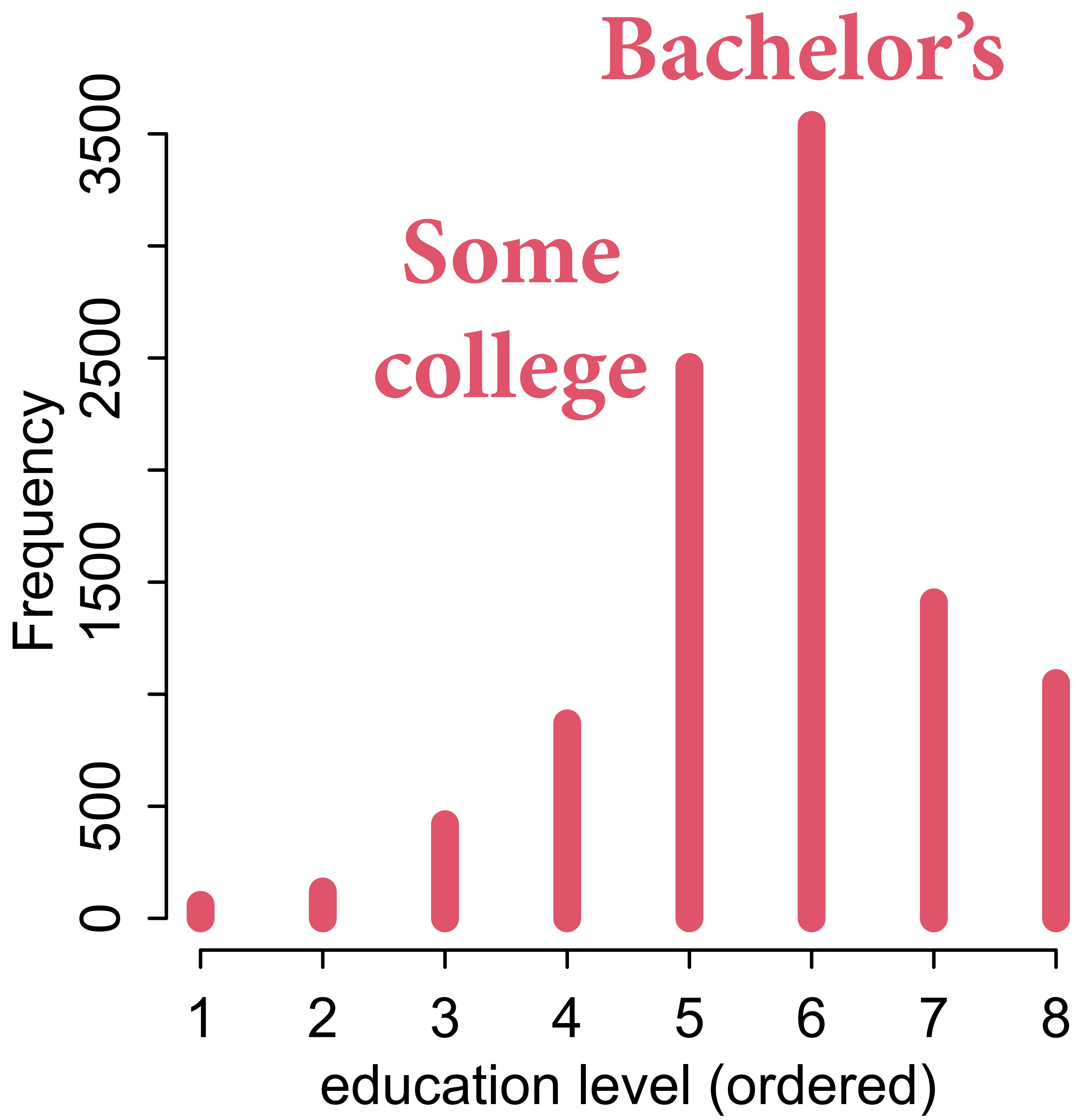
Ordered monotonic predictors

Education is an ordered category

Unlikely that each level has same effect

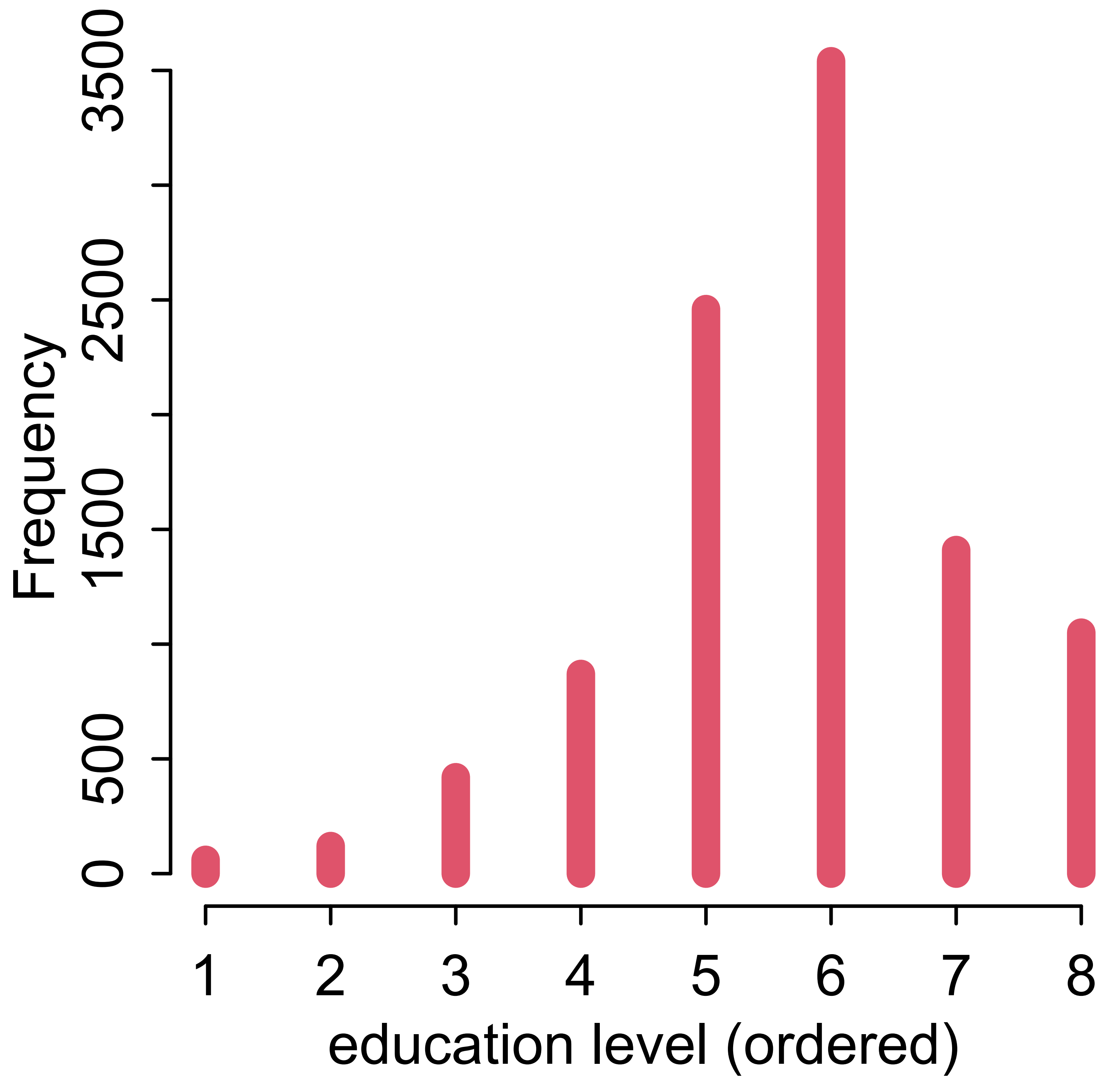
Want a parameter for each level

But how to enforce ordering, so that each level has larger (or smaller) effect than previous?



Ordered monotonic predictors

1 (elementary)	$\phi_i = 0$
2 (middle school)	$\phi_i = \delta_1$
3 (some high school)	$\phi_i = \delta_1 + \delta_2$
4 (high school)	$\phi_i = \delta_1 + \delta_2 + \delta_3$
5 (some college)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4$
6 (college)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5$
7 (master's)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6$
8 (doctorate)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7$



Ordered monotonic predictors

1 (elementary)	$\phi_i = 0$
2 (middle school)	$\phi_i = \delta_1$
3 (some high school)	$\phi_i = \delta_1 + \delta_2$
4 (high school)	$\phi_i = \delta_1 + \delta_2 + \delta_3$
5 (some college)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4$
6 (college)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5$
7 (master's)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6$
8 (doctorate)	$\phi_i = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 = \boxed{\beta_E}$

maximum effect of education

Ordered monotonic predictors

1 (elementary)

2 (middle school)

3 (some high school)

4 (high school)

5 (some college)

6 (college)

7 (master's)

8 (doctorate)

$$\delta_0 = 0$$

$$\sum_{j=0}^{7} \delta_j = 1$$

Ordered monotonic predictors

1 (elementary)

2 (middle school)

3 (some high school)

4 (high school)

5 (some college)

6 (college)

7 (master's)

8 (doctorate)

education level

$$\phi_i = \beta_E \sum_{j=0}^{E_i - 1} \delta_j$$

maximum effect

proportion of maximum effect

Ordered monotonic *priors*

How do we set priors for the delta parameters?

delta parameters form a simplex

Simplex: vector that sums to 1

$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_E \sum_{j=0}^{E_i - 1} \delta_j + \ldots$$

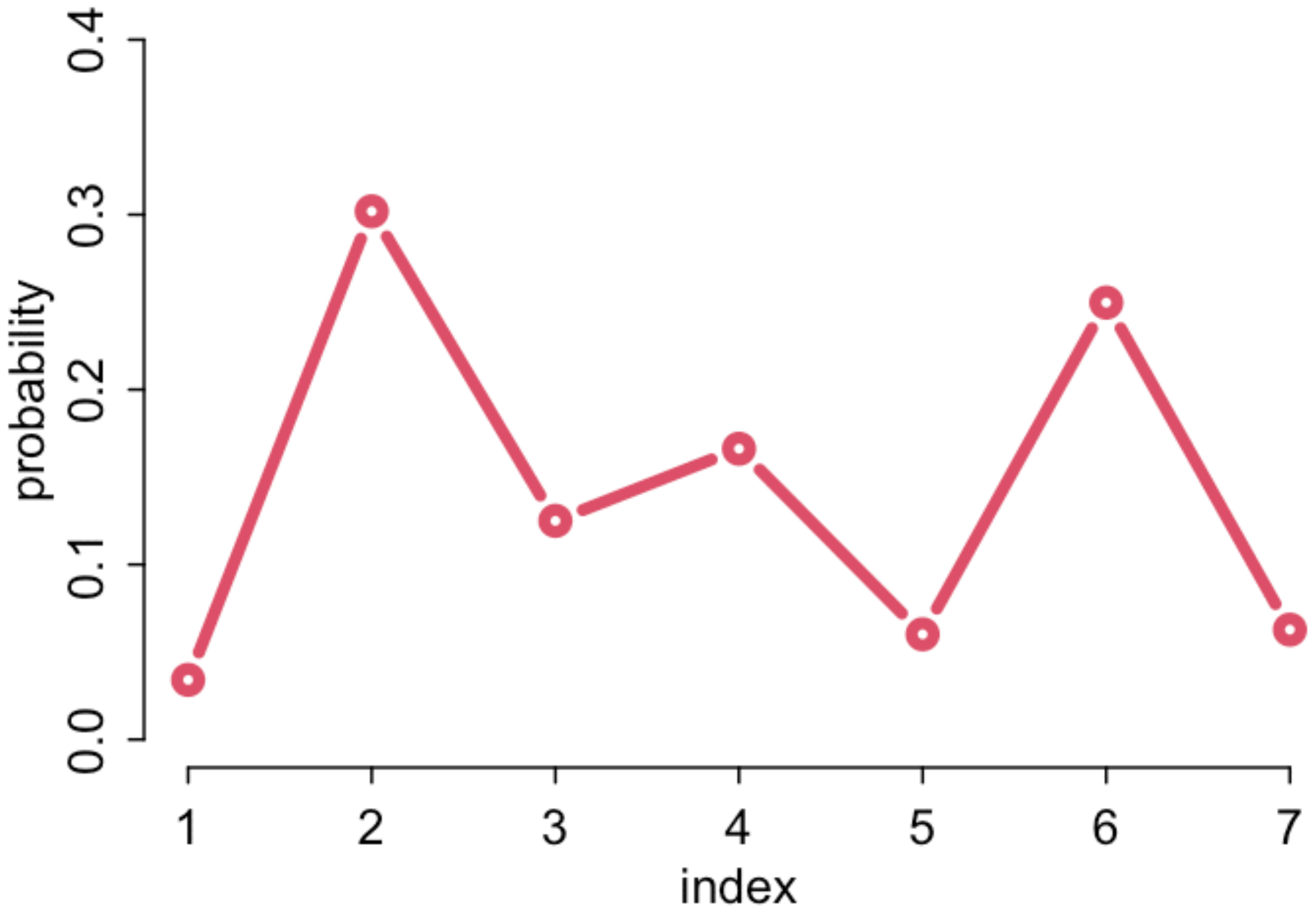
$$\alpha_j \sim \text{Normal}(0,1)$$

$$\beta\_ \sim \text{Normal}(0,0.5)$$

$$\delta_j \sim ?$$

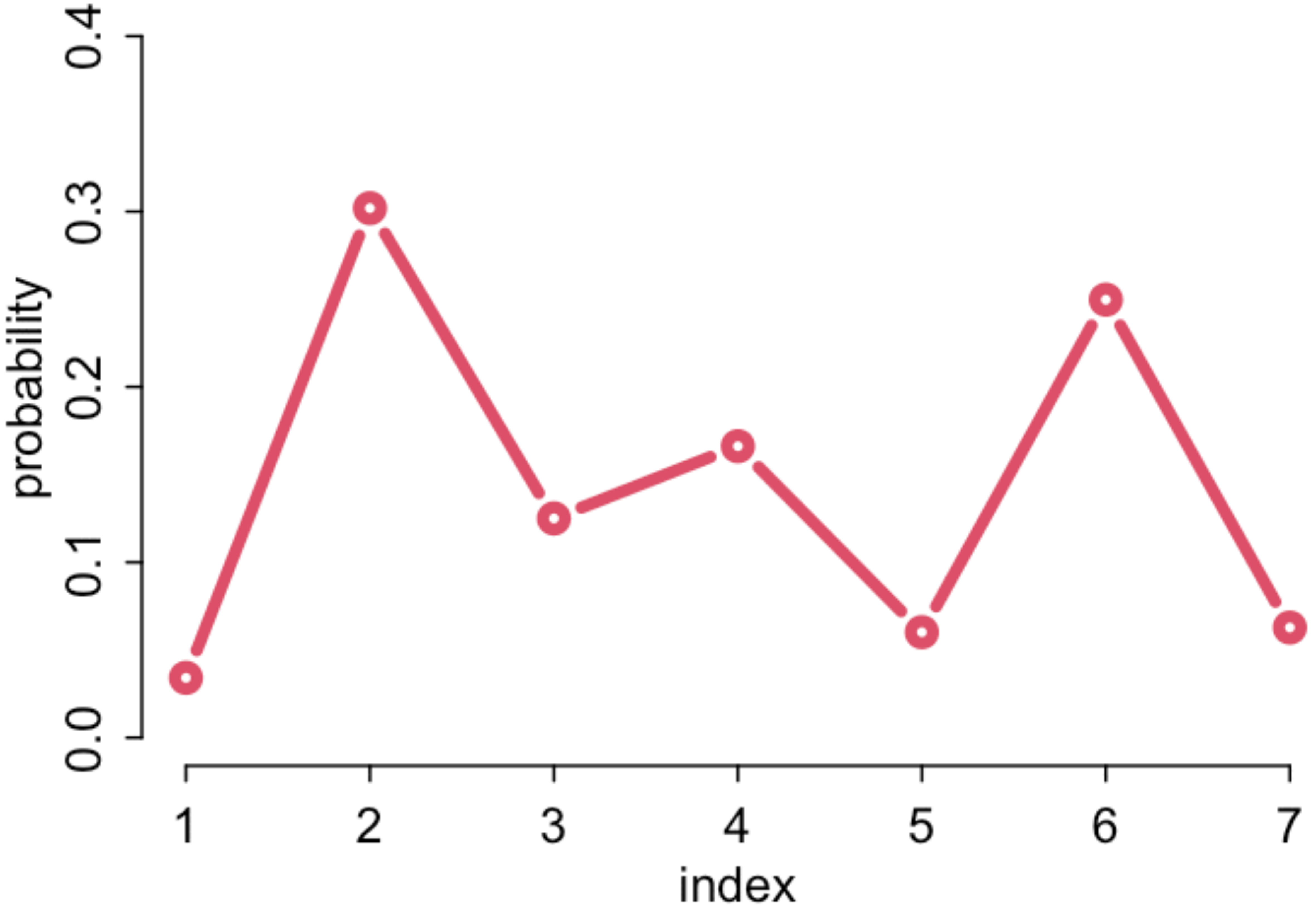
$$\delta \sim \text{Dirichlet}(a)$$

$$a = [2,2,2,2,2,2,2]$$



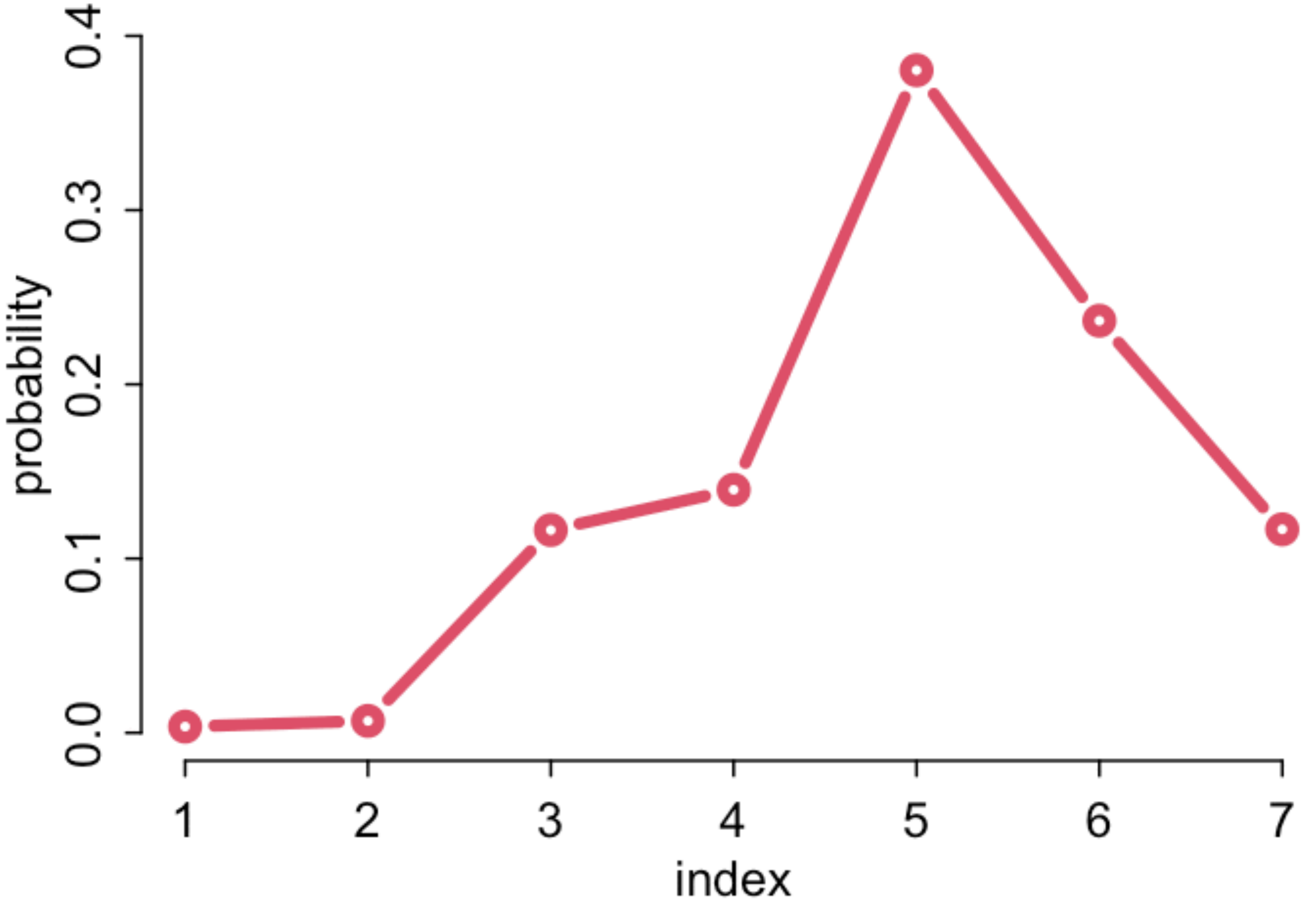
$\delta \sim \text{Dirichlet}(a)$

$a = [2,2,2,2,2,2,2]$



$\delta \sim \text{Dirichlet}(a)$

$a = [1,2,3,4,5,6,7]$



```r
edu_levels <- c( 6 , 1 , 8 , 4 , 7 , 2 , 5 , 3 )
edu_new <- edu_levels[ d$edu ]

dat$E <- edu_new
dat$a <- rep(2,7) # dirichlet prior

mRXE <- ulam(
    alist(

        R ~ ordered_logistic( phi , alpha ),
        phi <- bE*sum( delta_j[1:E] ) +
               bA*A + bI*I + bC*C,
        alpha ~ normal( 0 , 1 ),
        c(bA,bI,bC,bE) ~ normal( 0 , 0.5 ),
        vector[8]: delta_j <<- append_row( 0 , delta ),
        simplex[7]: delta ~ dirichlet( a )

    ), data=dat , chains=4 , cores=4 )
```

$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_E \sum_{j=0}^{E_i - 1} \delta_j + \ldots$$

$$\alpha_j \sim \text{Normal}(0,1)$$

$$\beta\_ \sim \text{Normal}(0,0.5)$$

$$\delta \sim \text{Dirichlet}(a)$$

```r
edu_levels <- c( 6 , 1 , 8 , 4 , 7 , 2 , 5 , 3 )
edu_new <- edu_levels[ d$edu ]

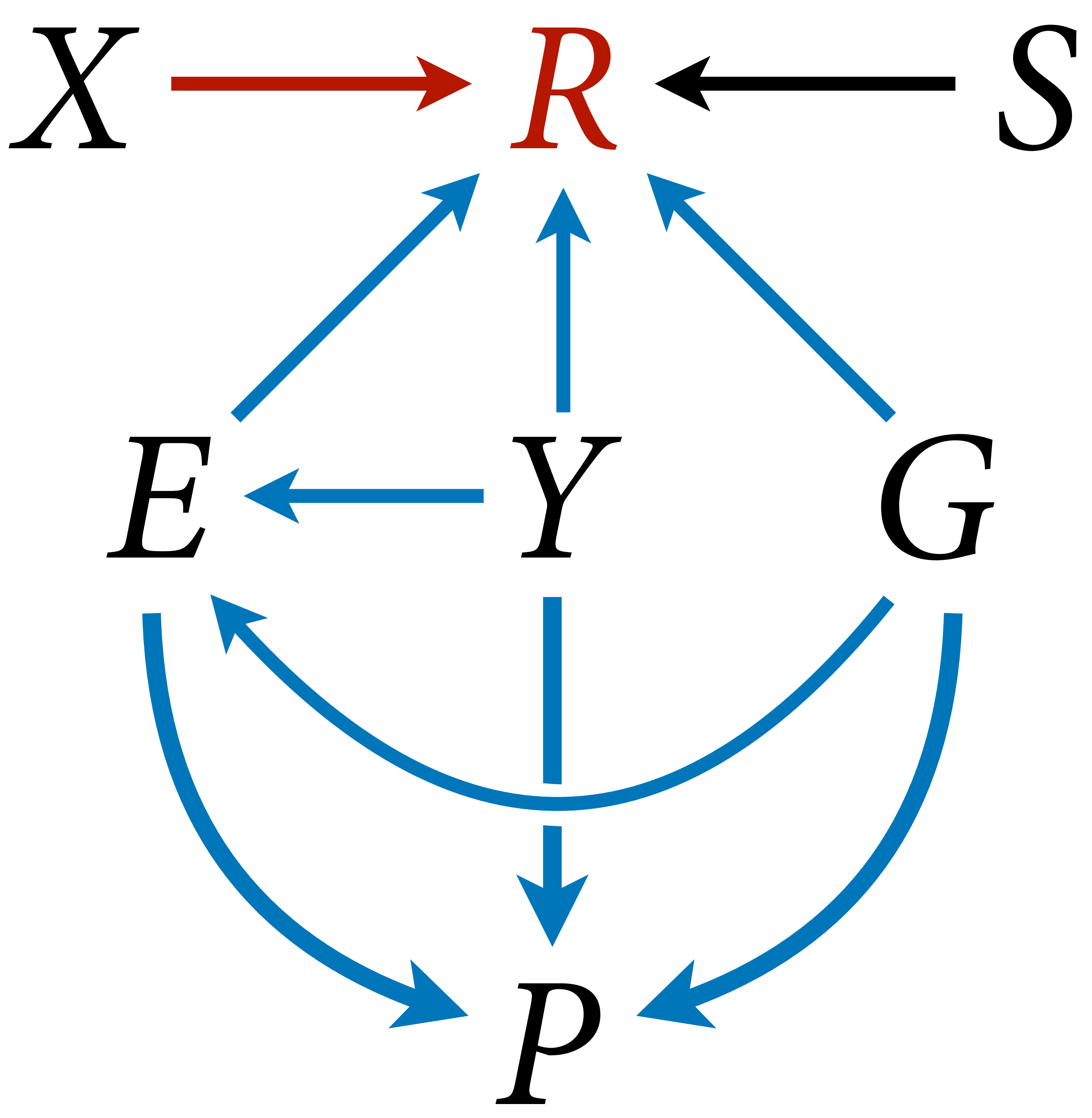
dat$E <- edu_new
dat$a <- rep(2,7) # dirichlet prior

mRXE <- ulam(
    alist(

        R ~ ordered_logistic( phi , alpha ),
        phi <- bE*sum( delta_j[1:E] ) +
               bA*A + bI*I + bC*C,
        alpha ~ normal( 0 , 1 ),
        c(bA,bI,bC,bE) ~ normal( 0 , 0.5 ),
        vector[8]: delta_j <<- append_row( 0 ,
        simplex[7]: delta ~ dirichlet( a )

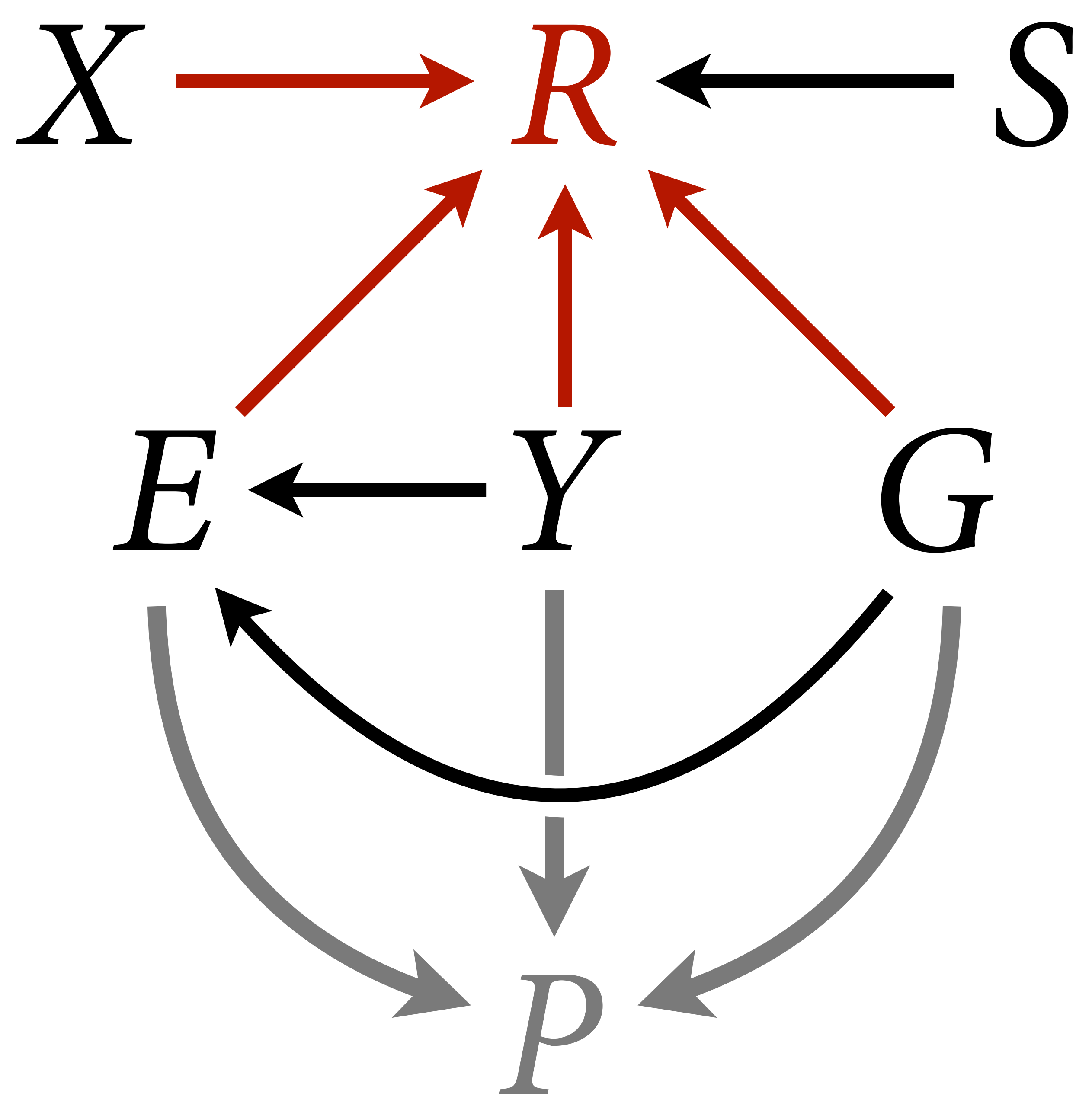
    ), data=dat , chains=4 , cores=4 )
```

```
> precis(mRXE,2)
           mean   sd  5.5% 94.5% n_eff Rhat4
alpha[1] -3.07 0.14 -3.32 -2.86   793     1
alpha[2] -2.39 0.14 -2.63 -2.17   804     1
alpha[3] -1.81 0.14 -2.05 -1.60   811     1
alpha[4] -0.79 0.14 -1.03 -0.57   799     1
alpha[5] -0.12 0.14 -0.36  0.10   804     1
alpha[6]  0.79 0.14  0.54  1.00   831     1
bE       -0.31 0.16 -0.57 -0.06   838     1
bC       -0.96 0.05 -1.04 -0.88  1757     1
bI       -0.72 0.04 -0.77 -0.66  1982     1
bA       -0.70 0.04 -0.77 -0.64  1779     1
delta[1]  0.22 0.13  0.05  0.47  1227     1
delta[2]  0.14 0.09  0.03  0.31  2258     1
delta[3]  0.20 0.11  0.05  0.38  2256     1
delta[4]  0.17 0.09  0.04  0.34  1926     1
delta[5]  0.04 0.05  0.01  0.12   945     1
delta[6]  0.10 0.07  0.02  0.23  1870     1
delta[7]  0.13 0.08  0.03  0.27  2335     1
```



bE not interpretable

```
> precis(mRXE,2)
            mean   sd  5.5% 94.5% n_eff Rhat4
alpha[1] -3.07 0.14 -3.32 -2.86   793     1
alpha[2] -2.39 0.14 -2.63 -2.17   804     1
alpha[3] -1.81 0.14 -2.05 -1.60   811     1
alpha[4] -0.79 0.14 -1.03 -0.57   799     1
alpha[5] -0.12 0.14 -0.36  0.10   804     1
alpha[6]  0.79 0.14  0.54  1.00   831     1
bE       -0.31 0.16 -0.57 -0.06   838     1
bC       -0.96 0.05 -1.04 -0.88  1757     1
bI       -0.72 0.04 -0.77 -0.66  1982     1
bA       -0.70 0.04 -0.77 -0.64  1779     1
delta[1]  0.22 0.13  0.05  0.47  1227     1
delta[2]  0.14 0.09  0.03  0.31  2258     1
delta[3]  0.20 0.11  0.05  0.38  2256     1
delta[4]  0.17 0.09  0.04  0.34  1926     1
delta[5]  0.04 0.05  0.01  0.12   945     1
delta[6]  0.10 0.07  0.02  0.23  1870     1
delta[7]  0.13 0.08  0.03  0.27  2335     1
```



$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_{E,G[i]} \sum_{j=0}^{E_i-1} \delta_j + \beta_{A,G[i]} A_i + \beta_{I,G[i]} I_i + \beta_{C,G[i]} C_i + \beta_{Y,G[i]} Y_i$$

$$\alpha_j \sim \text{Normal}(0,1)$$

$$\beta_{\_} \sim \text{Normal}(0,0.5)$$

$$\delta \sim \text{Dirichlet}(a)$$

```
dat$Y <- standardize(d$age)

mRXEYGt <- ulam(
    alist(
        R ~ ordered_logistic( phi , alpha ),
        phi <- bE[G]*sum( delta_j[1:E] ) +
               bA[G]*A + bI[G]*I + bC[G]*C +
               bY[G]*Y,
        alpha ~ normal( 0 , 1 ),
        bA[G] ~ normal( 0 , 0.5 ),
        bI[G] ~ normal( 0 , 0.5 ),
        bC[G] ~ normal( 0 , 0.5 ),
        bE[G] ~ normal( 0 , 0.5 ),
        bY[G] ~ normal( 0 , 0.5 ),
        vector[8]: delta_j <<- append_row( 0 , delta ),
        simplex[7]: delta ~ dirichlet( a )
    ), data=dat , chains=4 , cores=4 , threads=2 )
```

$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_{E,G[i]} \sum_{j=0}^{E_i-1} \delta_j + \beta_{A,G[i],i} A_i + \beta_{I,G[i]} I_i + \beta_{C,G[i]} C_i + \beta_{Y,G[i]} Y_i$$

$$\alpha_j \sim \text{Normal}(0,1)$$

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$$\delta \sim \text{Dirichlet}(a)$$

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    alist(
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        phi <- bE[G]*sum( delta_j[1:E] ) +
               bA[G]*A + bI[G]*I + bC[G]*C +
               bY[G]*Y,
        alpha ~ normal( 0 , 1 ),
        bA[G] ~ normal( 0 , 0.5 ),
        bI[G] ~ normal( 0 , 0.5 ),
        bC[G] ~ normal( 0 , 0.5 ),
        bE[G] ~ normal( 0 , 0.5 ),
        bY[G] ~ normal( 0 , 0.5 ),
        vector[8]: delta_j <<- append_row( 0 , delta ),
        simplex[7]: delta ~ dirichlet( a )
    ), data=dat , chains=4 , cores=4 , threads=2 )
```

4 chains times 2 threads each = 8 cores

$$R_i \sim \text{OrderedLogit}(\phi_i, \alpha)$$

$$\phi_i = \beta_{E,G[i]} \sum_{j=0}^{E_i-1} \delta_j + \beta_{A,G[i],i} A_i + \beta_{I,G[i]} I_i + \beta_{C,G[i]} C_i + \beta_{Y,G[i]} Y_i$$

$$\alpha_j \sim \text{Normal}(0,1)$$

$$\beta_{\_} \sim \text{Normal}(0,0.5)$$

$$\delta \sim \text{Dirichlet}(a)$$

```
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mRXEYGt <- ulam(
    alist(
        R ~ ordered_logistic( phi , alpha ),
        phi <- bE[G]*sum( delta_j[1:E] ) +
               bA[G]*A + bI[G]*I + bC[G]*C +
               bY[G]*Y,
        alpha ~ normal( 0 , 1 ),
        bA[G] ~ normal( 0 , 0.5 ),
        bI[G] ~ normal( 0 , 0.5 ),
        bC[G] ~ normal( 0 , 0.5 ),
        bE[G] ~ normal( 0 , 0.5 ),
        bY[G] ~ normal( 0 , 0.5 ),
        vector[8]: delta_j <<- append_row( 0 , delta ),
        simplex[7]: delta ~ dirichlet( a )
    ), data=dat , chains=4 , cores=4 , threads=2 )
```

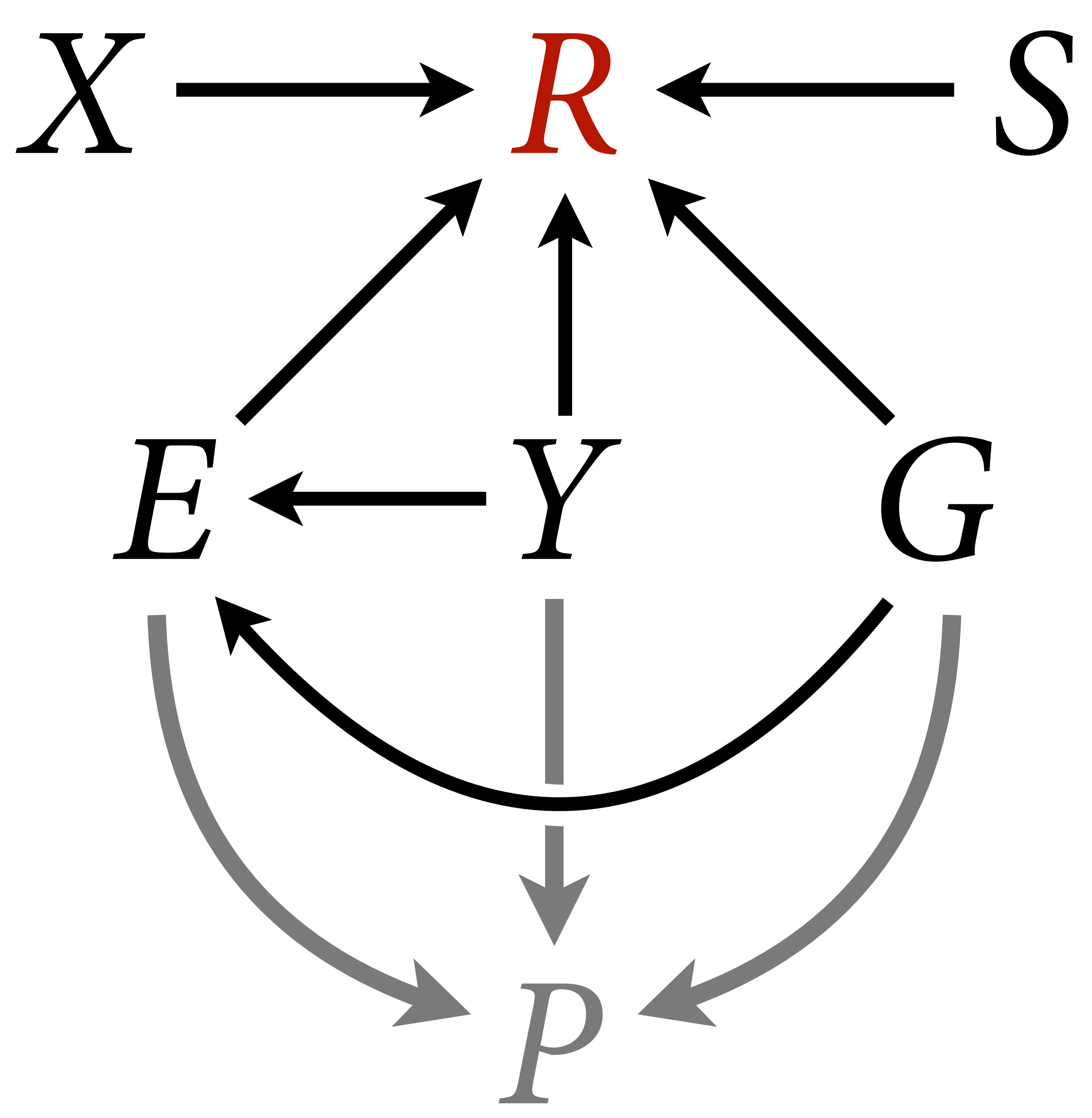
4 chains times 2 threads each = 8 cores

1 thread each

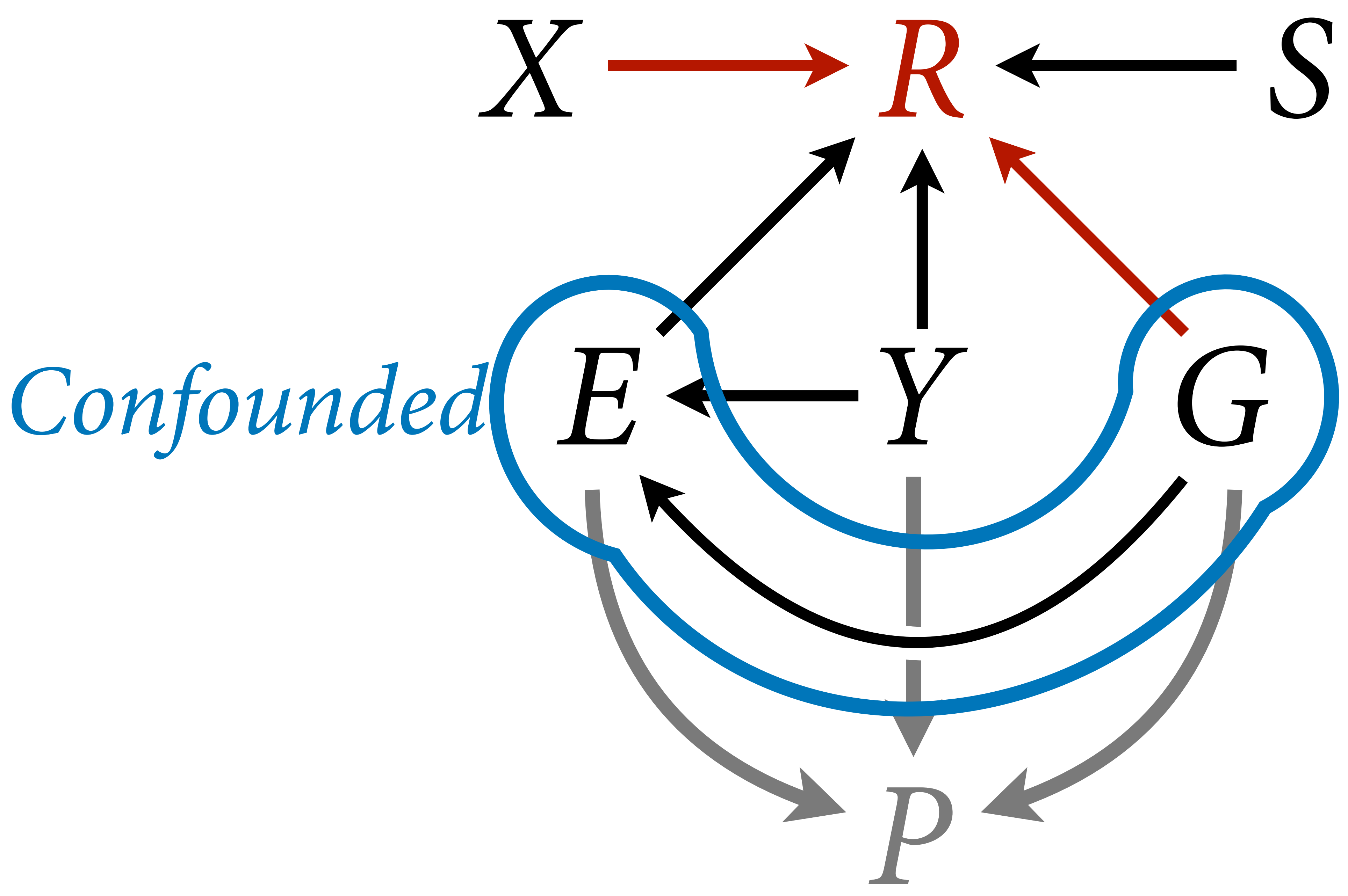
```
Sampling durations (minutes):
        warmup sample total
chain:1   6.53   3.99 10.52
chain:2   7.33   2.66  9.99
chain:3   6.88   3.70 10.58
chain:4   6.40   2.63  9.03
```

2 threads each

```
Sampling durations (minutes):
        warmup sample total
chain:1   4.41   1.80  6.21
chain:2   4.69   1.87  6.56
chain:3   5.14   1.56  6.70
chain:4   4.21   1.84  6.05
```

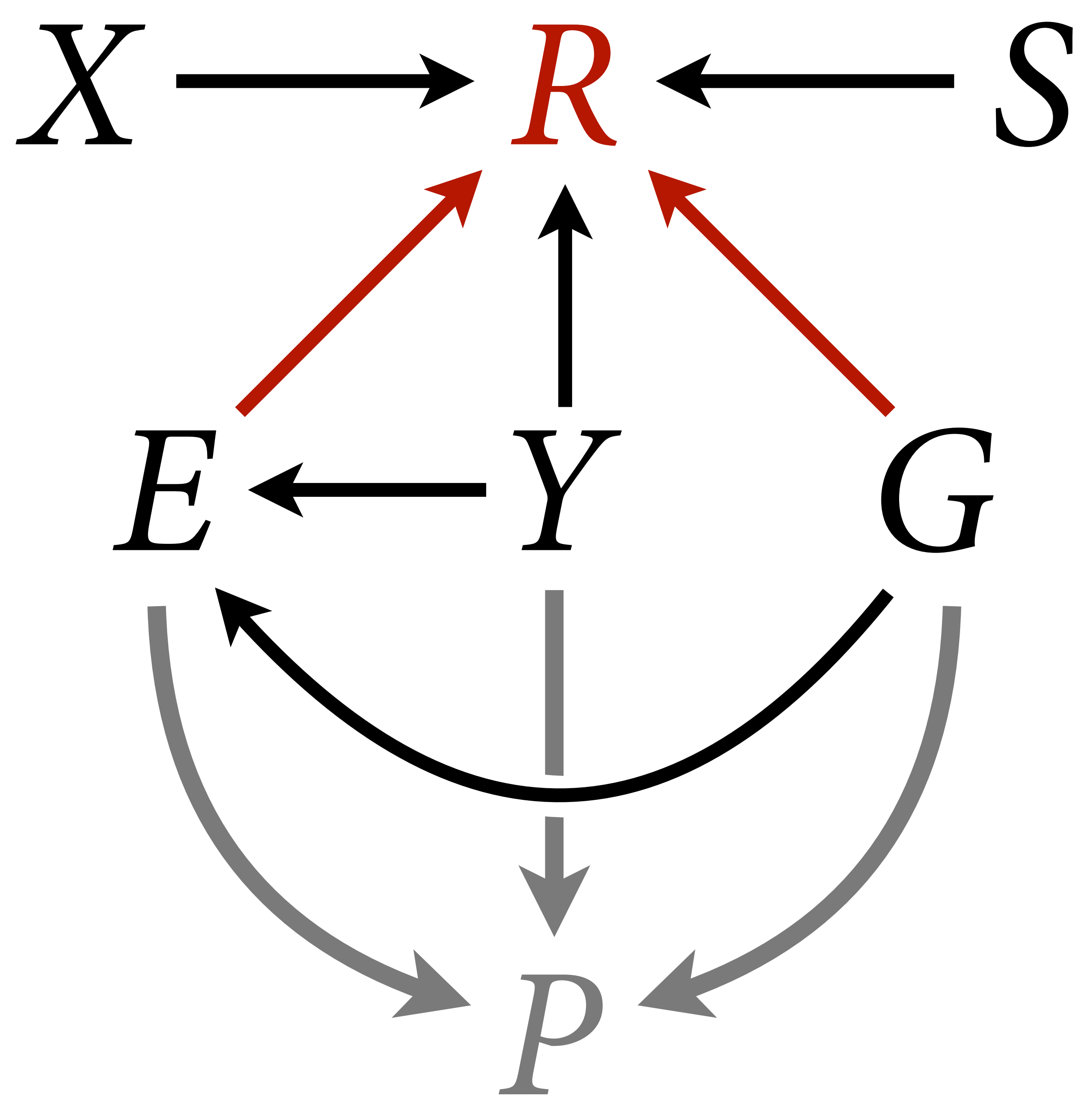



```
> precis(mRXEYGt,2)
          mean   sd  5.5% 94.5% n_eff Rhat4
alpha[1] -2.89 0.10 -3.06 -2.73   729     1
alpha[2] -2.21 0.10 -2.37 -2.06   728     1
alpha[3] -1.62 0.10 -1.78 -1.47   724     1
alpha[4] -0.58 0.10 -0.74 -0.43   729     1
alpha[5]  0.11 0.10 -0.05  0.26   726     1
alpha[6]  1.03 0.10  0.87  1.18   746     1
bA[1]    -0.56 0.06 -0.65 -0.47  1932     1
bA[2]    -0.81 0.05 -0.90 -0.73  2013     1
bI[1]    -0.66 0.05 -0.74 -0.58  2539     1
bI[2]    -0.76 0.05 -0.84 -0.68  2283     1
bC[1]    -0.77 0.07 -0.88 -0.65  2029     1
bC[2]    -1.09 0.07 -1.20 -0.99  2012     1
bE[1]    -0.63 0.14 -0.85 -0.42   810     1
bE[2]     0.41 0.14  0.19  0.62   795     1
bY[1]     0.00 0.03 -0.05  0.05  2740     1
bY[2]    -0.13 0.03 -0.18 -0.09  1426     1
delta[1]  0.15 0.08  0.04  0.31  1759     1
delta[2]  0.15 0.09  0.04  0.30  2440     1
delta[3]  0.29 0.11  0.11  0.46  2001     1
delta[4]  0.08 0.05  0.02  0.17  2414     1
delta[5]  0.06 0.04  0.01  0.14  1087     1
delta[6]  0.24 0.07  0.13  0.34  2301     1
delta[7]  0.04 0.02  0.01  0.08  2755     1
```



Only direct effect G

```
> precis(mRXEYGt,2)
            mean   sd  5.5% 94.5% n_eff Rhat4
alpha[1]   -2.89 0.10 -3.06 -2.73   729     1
alpha[2]   -2.21 0.10 -2.37 -2.06   728     1
alpha[3]   -1.62 0.10 -1.78 -1.47   724     1
alpha[4]   -0.58 0.10 -0.74 -0.43   729     1
alpha[5]    0.11 0.10 -0.05  0.26   726     1
alpha[6]    1.03 0.10  0.87  1.18   746     1
bA[1]      -0.56 0.06 -0.65 -0.47  1932     1
bA[2]      -0.81 0.05 -0.90 -0.73  2013     1
bI[1]      -0.66 0.05 -0.74 -0.58  2539     1
bI[2]      -0.76 0.05 -0.84 -0.68  2283     1
bC[1]      -0.77 0.07 -0.88 -0.65  2029     1
bC[2]      -1.09 0.07 -1.20 -0.99  2012     1
bE[1]      -0.63 0.14 -0.85 -0.42   810     1
bE[2]       0.41 0.14  0.19  0.62   795     1
bY[1]       0.00 0.03 -0.05  0.05  2740     1
bY[2]      -0.13 0.03 -0.18 -0.09  1426     1
delta[1]    0.15 0.08  0.04  0.31  1759     1
delta[2]    0.15 0.09  0.04  0.30  2440     1
delta[3]    0.29 0.11  0.11  0.46  2001     1
delta[4]    0.08 0.05  0.02  0.17  2414     1
delta[5]    0.06 0.04  0.01  0.14  1087     1
delta[6]    0.24 0.07  0.13  0.34  2301     1
delta[7]    0.04 0.02  0.01  0.08  2755     1
```




```
> precis(mRXEYGt,2)
           mean   sd  5.5% 94.5% n_eff Rhat4
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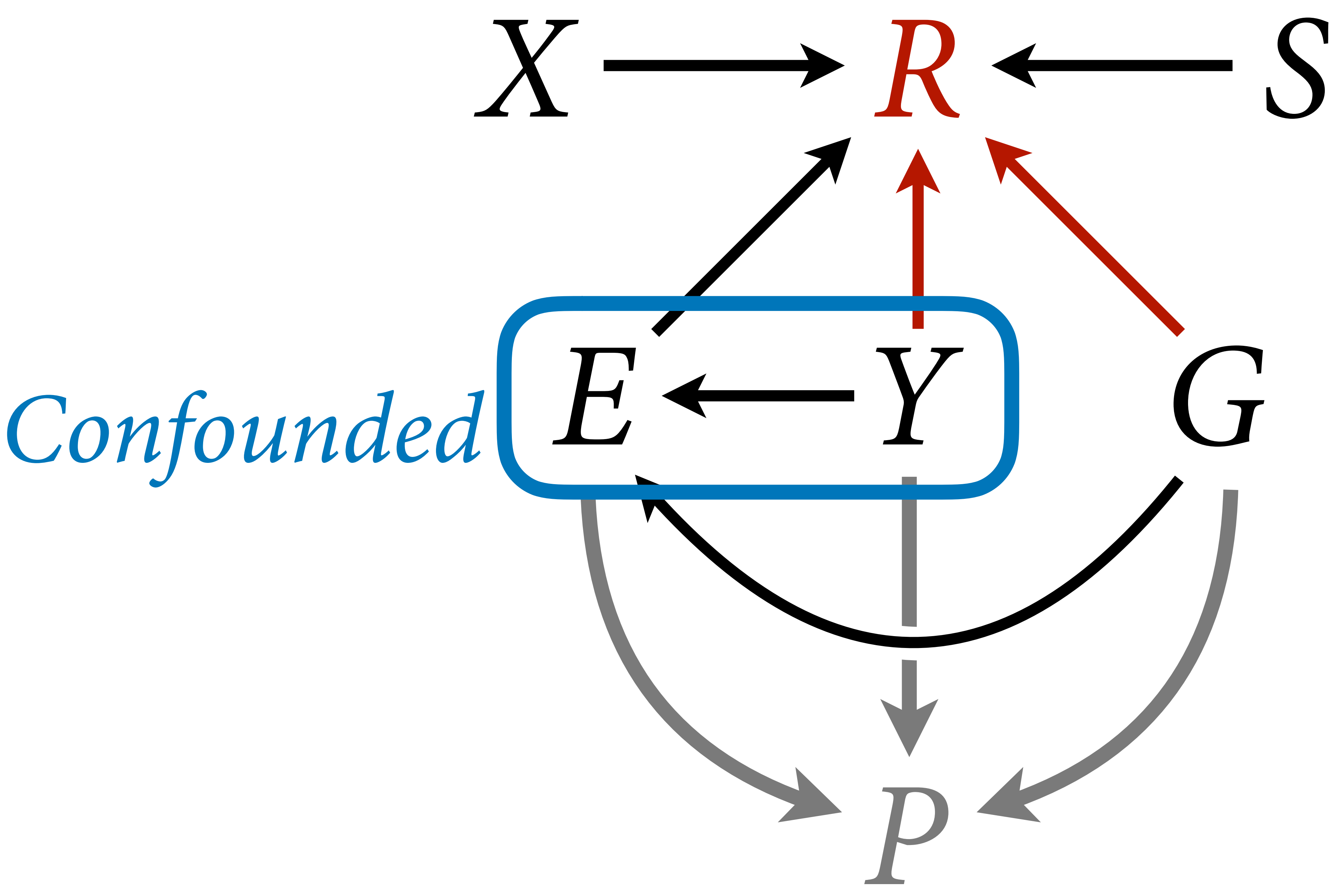
alpha[3] -1.62 0.10 -1.78 -1.47   724     1
alpha[4] -0.58 0.10 -0.74 -0.43   729     1
alpha[5]  0.11 0.10 -0.05  0.26   726     1
alpha[6]  1.03 0.10  0.87  1.18   746     1
bA[1]    -0.56 0.06 -0.65 -0.47  1932     1
bA[2]    -0.81 0.05 -0.90 -0.73  2013     1
bI[1]    -0.66 0.05 -0.74 -0.58  2539     1
bI[2]    -0.76 0.05 -0.84 -0.68  2283     1
bC[1]    -0.77 0.07 -0.88 -0.65  2029     1
bC[2]    -1.09 0.07 -1.20 -0.99  2012     1
bE[1]    -0.63 0.14 -0.85 -0.42   810     1
bE[2]     0.41 0.14  0.19  0.62   795     1
bY[1]     0.00 0.03 -0.05  0.05  2740     1
bY[2]    -0.13 0.03 -0.18 -0.09  1426     1
delta[1]  0.15 0.08  0.04  0.31  1759     1
delta[2]  0.15 0.09  0.04  0.30  2440     1
delta[3]  0.29 0.11  0.11  0.46  2001     1
delta[4]  0.08 0.05  0.02  0.17  2414     1
delta[5]  0.06 0.04  0.01  0.14  1087     1
delta[6]  0.24 0.07  0.13  0.34  2301     1
delta[7]  0.04 0.02  0.01  0.08  2755     1
```



Only direct effect

```
> precis(mRXEYGt,2)
           mean   sd  5.5% 94.5% n_eff Rhat4
alpha[1]  -2.89 0.10 -3.06 -2.73   729     1
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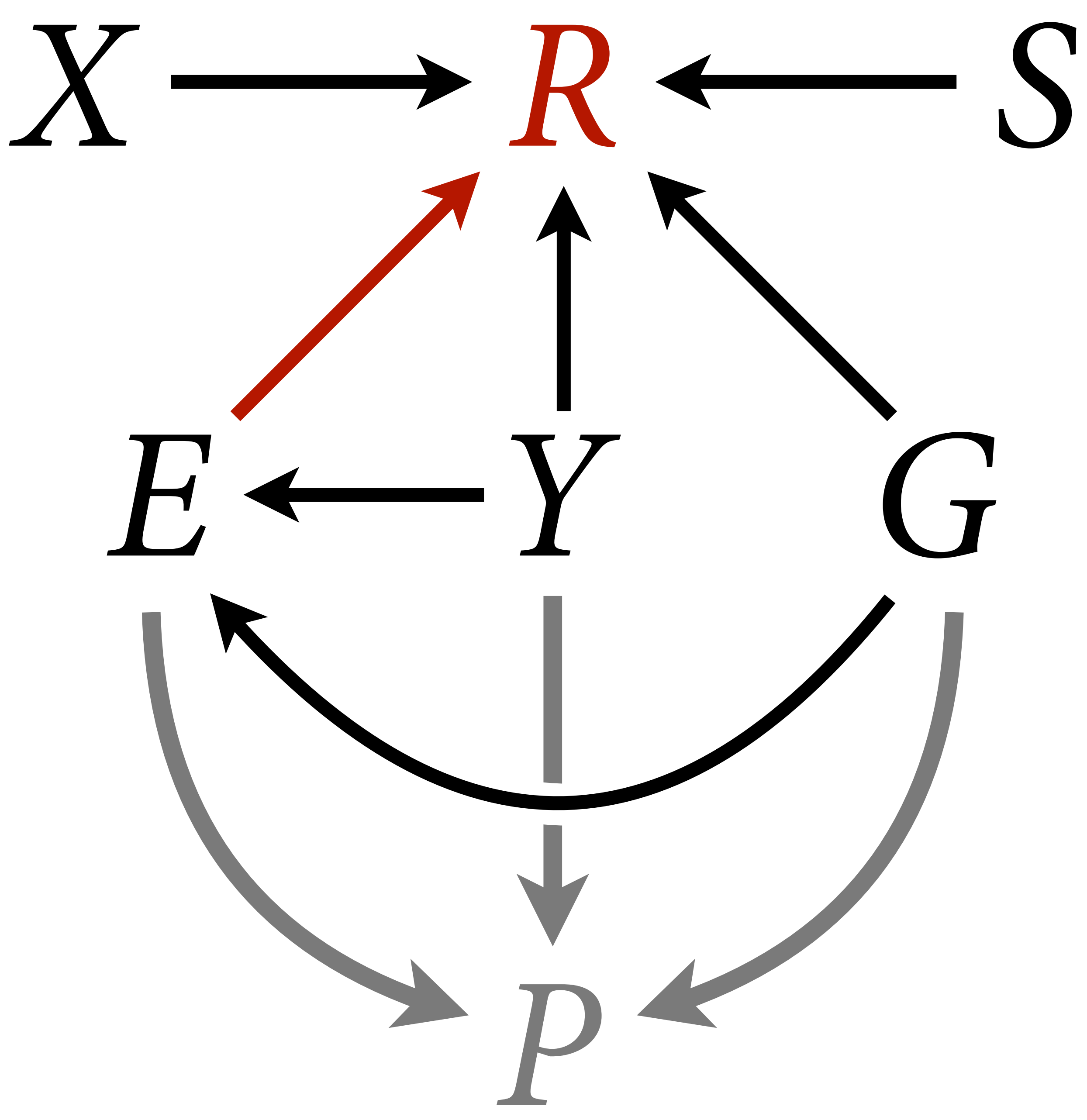
bA[1]     -0.56 0.06 -0.65 -0.47  1932     1
bA[2]     -0.81 0.05 -0.90 -0.73  2013     1
bI[1]     -0.66 0.05 -0.74 -0.58  2539     1
bI[2]     -0.76 0.05 -0.84 -0.68  2283     1
bC[1]     -0.77 0.07 -0.88 -0.65  2029     1
bC[2]     -1.09 0.07 -1.20 -0.99  2012     1
bE[1]     -0.63 0.14 -0.85 -0.42   810     1
bE[2]      0.41 0.14  0.19  0.62   795     1
bY[1]      0.00 0.03 -0.05  0.05  2740     1
bY[2]     -0.13 0.03 -0.18 -0.09  1426     1
delta[1]   0.15 0.08  0.04  0.31  1759     1
delta[2]   0.15 0.09  0.04  0.30  2440     1
delta[3]   0.29 0.11  0.11  0.46  2001     1
delta[4]   0.08 0.05  0.02  0.17  2414     1
delta[5]   0.06 0.04  0.01  0.14  1087     1
delta[6]   0.24 0.07  0.13  0.34  2301     1
delta[7]   0.04 0.02  0.01  0.08  2755     1
```




```
> precis(mRXEYGt,2)
            mean   sd  5.5% 94.5% n_eff Rhat4
alpha[1]   -2.89 0.10 -3.06 -2.73   729     1
alpha[2]   -2.21 0.10 -2.37 -2.06   728     1
alpha[3]   -1.62 0.10 -1.78 -1.47   724     1
alpha[4]   -0.58 0.10 -0.74 -0.43   729     1
alpha[5]    0.11 0.10 -0.05  0.26   726     1
alpha[6]    1.03 0.10  0.87  1.18   746     1
bA[1]      -0.56 0.06 -0.65 -0.47  1932     1
bA[2]      -0.81 0.05 -0.90 -0.73  2013     1
bI[1]      -0.66 0.05 -0.74 -0.58  2539     1
bI[2]      -0.76 0.05 -0.84 -0.68  2283     1
bC[1]      -0.77 0.07 -0.88 -0.65  2029     1
bC[2]      -1.09 0.07 -1.20 -0.99  2012     1
bE[1]      -0.63 0.14 -0.85 -0.42   810     1
bE[2]       0.41 0.14  0.19  0.62   795     1
bY[1]       0.00 0.03 -0.05  0.05  2740     1
bY[2]      -0.13 0.03 -0.18 -0.09  1426     1
delta[1]    0.15 0.08  0.04  0.31  1759     1
delta[2]    0.15 0.09  0.04  0.30  2440     1
delta[3]    0.29 0.11  0.11  0.46  2001     1
delta[4]    0.08 0.05  0.02  0.17  2414     1
delta[5]    0.06 0.04  0.01  0.14  1087     1
delta[6]    0.24 0.07  0.13  0.34  2301     1
delta[7]    0.04 0.02  0.01  0.08  2755     1
```

Model mRXEYG2t stratifies by G, in Lecture 11 script

Complex causal effects

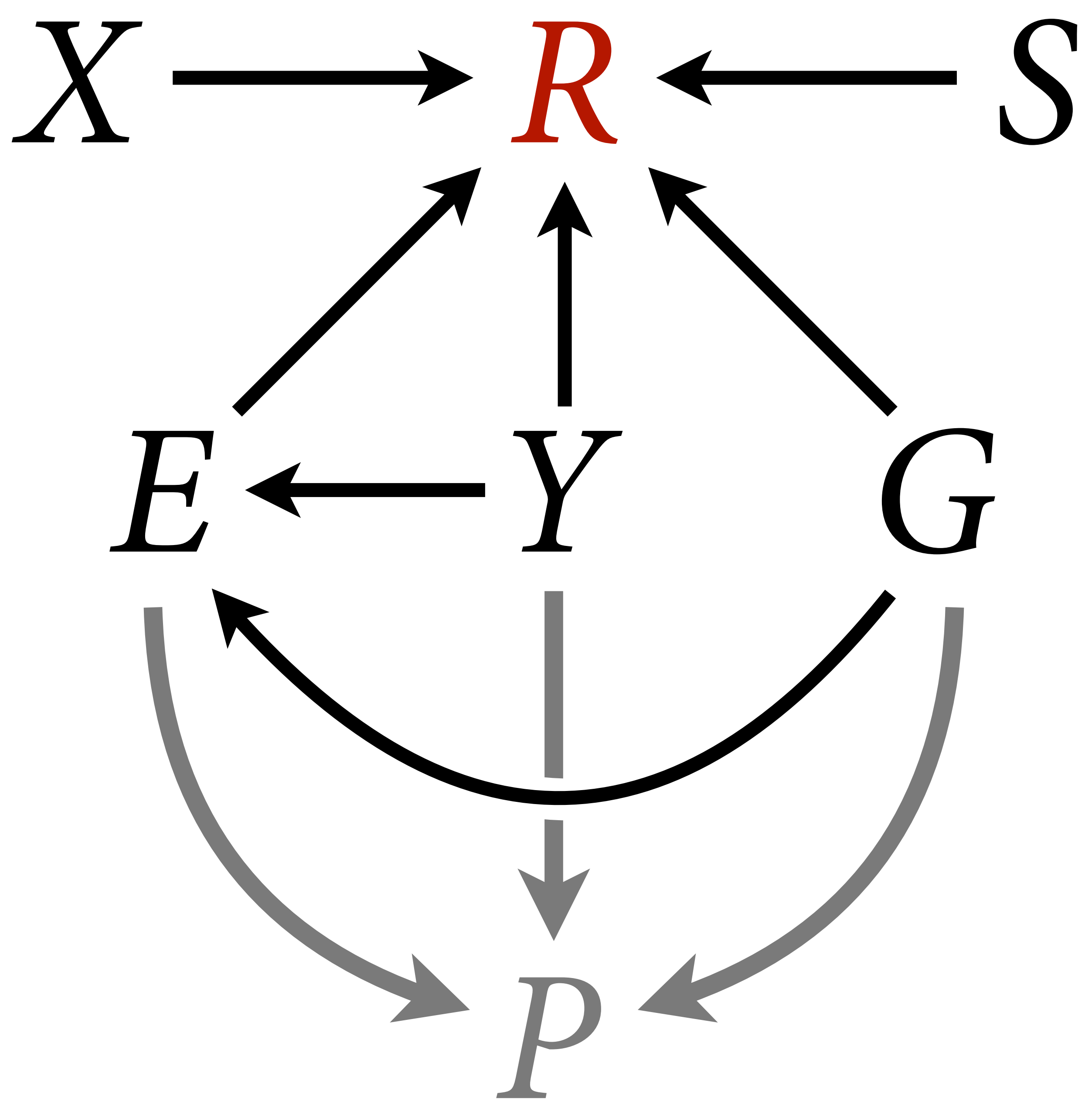
Causal effects (predicted consequences of intervention) require marginalization

Example: Causal effect of E requires distribution of Y and G to average over

Problem 1: Should not marginalize over this sample—*cursed P!* Post-stratify to new target.

Problem 2: Should not set all Y to same E

Example: Causal effect of Y requires effect of Y on E, which we cannot estimate (P again!)



Complex causal effects

Causal effects (predicted consequences of
interve

Examp
of Y an

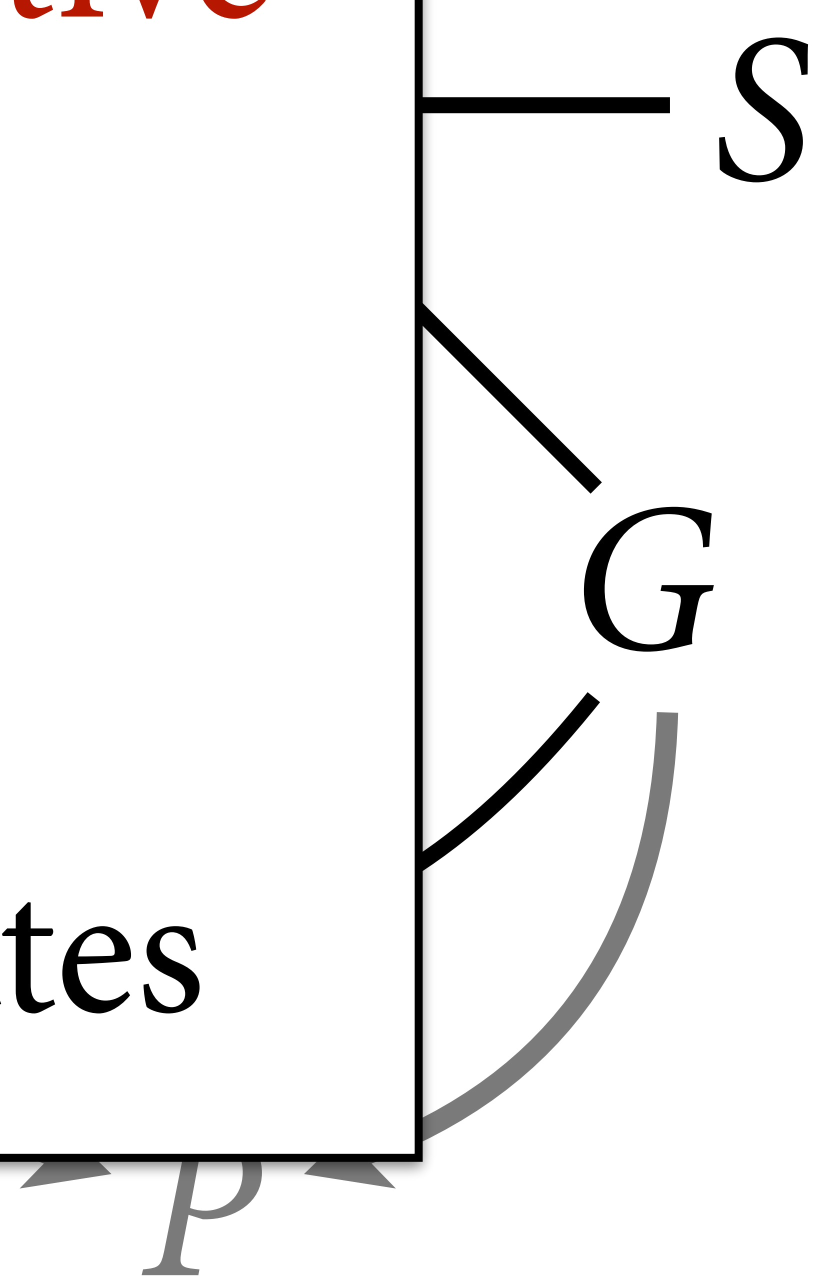
Proble
sample

Proble

No matter how complex, still just a generative simulation using posterior samples

Need generative model to plan estimation

Need generative model to compute estimates



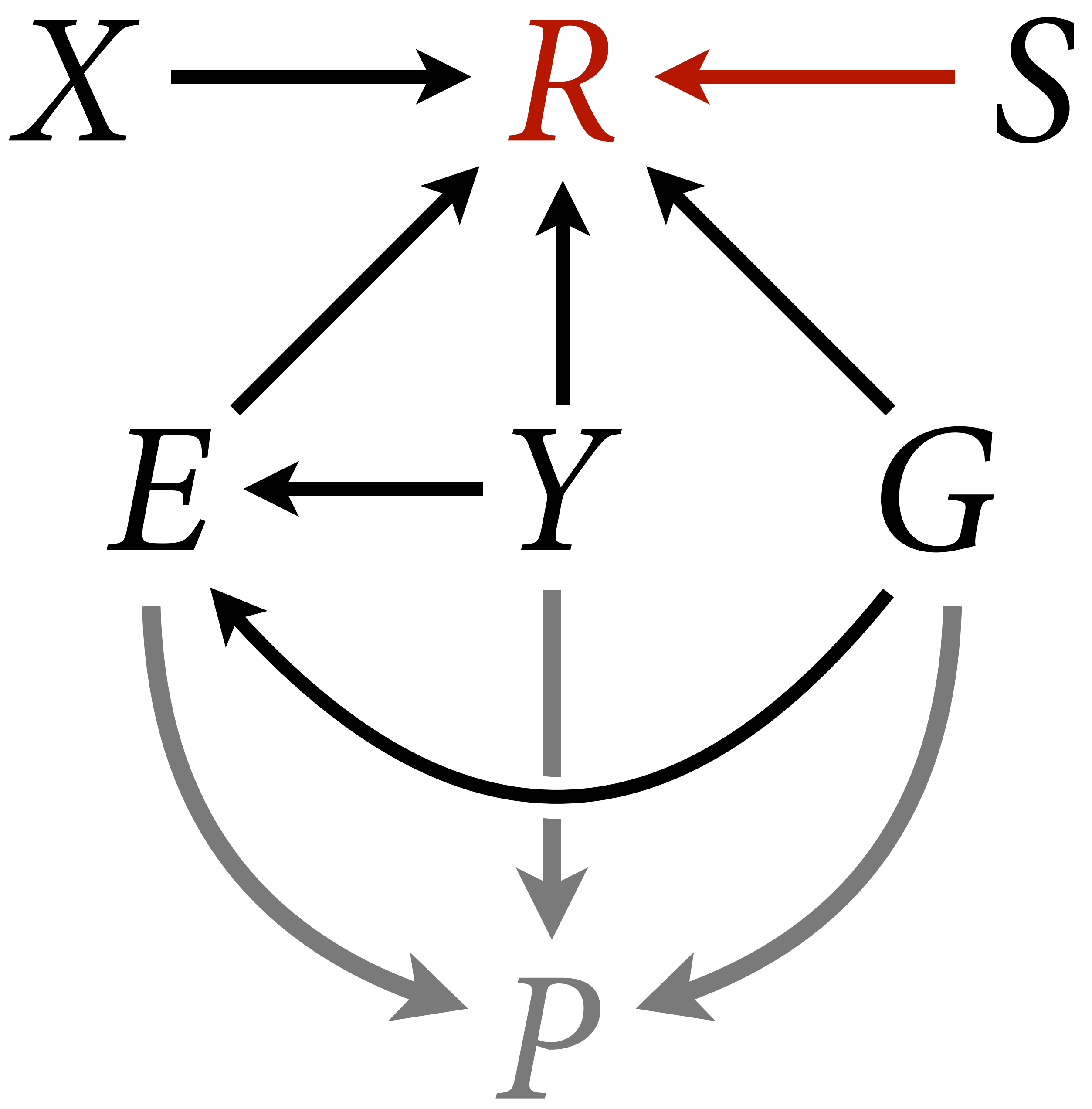
Example: Causal effect of Y requires effect of Y on E, which we cannot estimate (P again!)

Repeat observations

30 stories (S)

```
> table(d$story)

 aqu  boa  box  bur  car  che  pon  rub  sha  shi  spe  swi
 662  662 1324 1324  662  662  662  662  662  662  993  993
```



Repeat observations

30 stories (S)

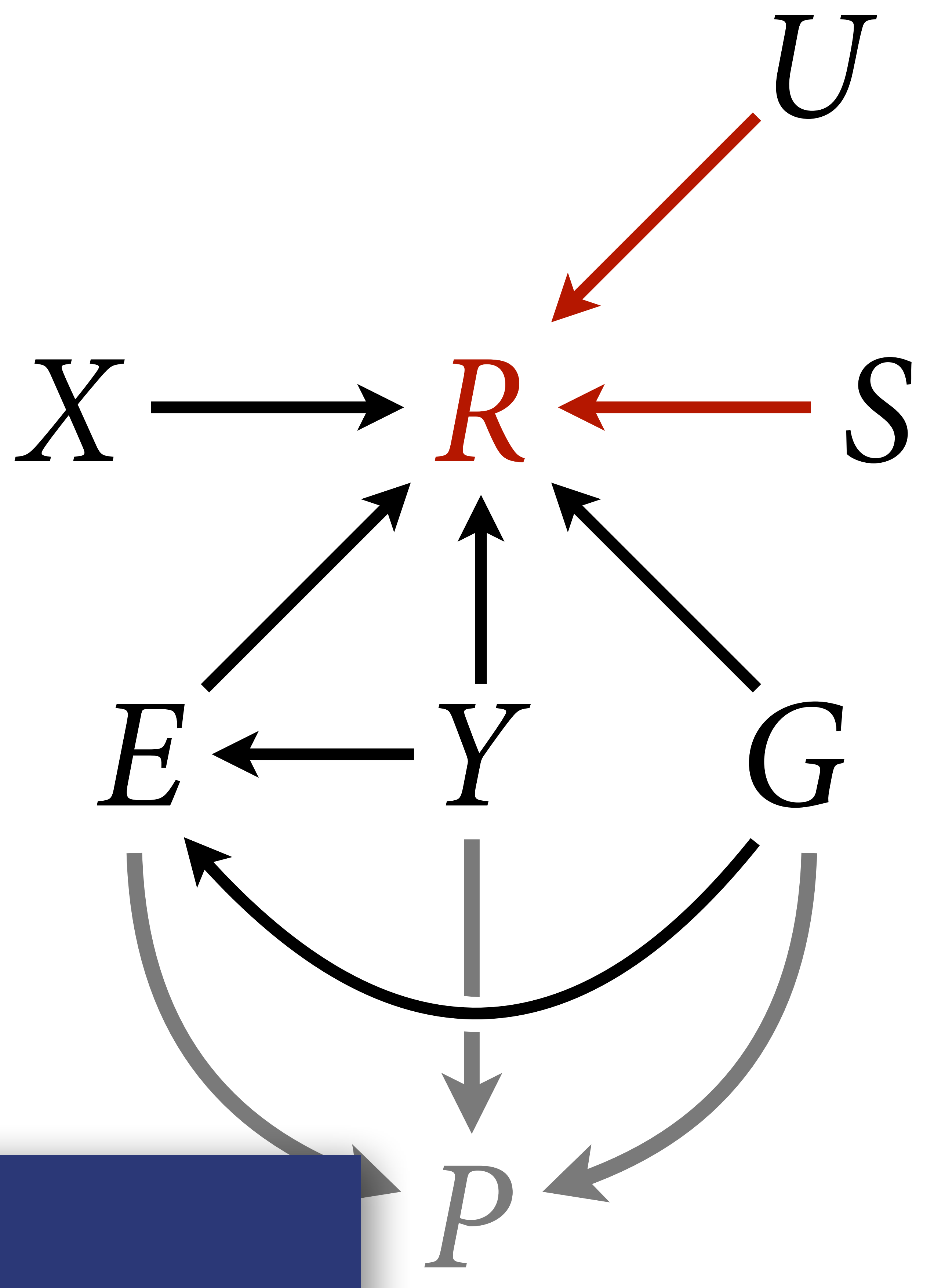
```
> table(d$story)

 aqu  boa  box  bur  car  che  pon  rub  sha  shi  spe  swi 
 662  662 1324 1324  662  662  662  662  662  662  993  993 
```

331 individuals (U)

```
> table(d$id)

96;434 96;445 96;451 96;456 96;458 96;466 96;467 96;474 96;480 96;481 96;497 
    30     30     30     30     30     30     30     30     30     30     30 
96;498 96;502 96;505 96;511 96;512 96;518 96;519 96;531 96;533 96;538 96;547 
    30     30     30     30     30     30     30     30     30     30     30 
96;550 96;553 96;555 96;558 96;560 96;562 96;566 96;570 96;581 96;586 96;591 
    30     30     30     30     30     30     30     30     30     30     30 
```



Course Schedule

Week 1	Bayesian inference	Chapters 1, 2, 3
Week 2	Linear models & Causal Inference	Chapter 4
Week 3	Causes, Confounds & Colliders	Chapters 5 & 6
Week 4	Overfitting / MCMC	Chapters 7, 8, 9
Week 5	Generalized Linear Models	Chapters 10, 11
Week 6	Ordered categories & Multilevel models	Chapters 12 & 13
Week 7	More Multilevel models	Chapters 13 & 14
Week 8	Multilevel models & Gaussian processes	Chapter 14
Week 9	Measurement & Missingness	Chapter 15
Week 10	Generalized Linear Madness	Chapter 16

https://github.com/rmcelreath/stat_rethinking_2022