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A simple mathematical model of society collapse applied to Easter Island

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Abstract. In this paper we consider a mathematical model for the evolution and collapse of the Easter Island society. Based on historical reports, the available primary resources consisted almost exclusively in the trees, then we describe the inhabitants and the resources as an isolated dynamical system. A mathematical, and numerical, analysis about the Easter Island community collapse is performed. In particular, we analyze the critical values of the fundamental parameters and a demographic curve is presented. The technological parameter, quantifying the exploitation of the resources, is calculated and applied to the case of another extinguished civilization (Copán Maya) confirming the consistency of the adopted model.
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Introduction

Easter Island history is a very famous example of an evolved human society that collapsed for overexploiting its fundamental resources [1–3] that in this case were essentially palm trees. They were covering the island [4] when, few dozens of individuals, first landed around 400 A.C. (After Christ). Its advanced culture was developed in isolation in a period of one thousand years approximately. Its ceremonial rituals and associated construction were demanding more and more natural resources especially endemic palm trees. The overexploitation of this kind of tree, very necessary as a primary resource (tool constructions, cooking fuel, erosion barrier, etc.) was related with the final collapse. Also diseases and contamination via non-endemic animal species carried involuntarily by the first inhabitants, such as rats, could have accelerated the environmental degradation of the island [5].

In this paper, a two-variable mathematical model concerning growing and collapse of this society is presented. Different from the usual Lotka-Volterra models [6, 7] where the carrying capacity variation comes from external natural forces, this work is directly connected with the population dynamics. Namely, population and carrying capacity are interacting dynamic variables.

The general mathematical treatment of a model describing such a complex society is a very hard task and probably not unique. Our aim is to settle the simplest model describing with acceptable precision the evolution of the Easterner society. With the idea of writing a model that could be generalized to a more complex system, we first divide the elements into two categories: the *resource* quantity R_i with $i = 1, 2 \cdots k$ and the *inhabitants* number (species) N_i with $i = 1, 2 \cdots m$. With the concept of resources we are meaning resources in a very large sense, it could be oil, trees, food and so on. The several kinds of resources are described by the index i. In a similar way, with the concept of inhabitants, we mean different species of animals or the internal subdivision of human beings in the country or in town or even in tribes. Leaving the idea of a constant quantity of resources that leads to the logistic equation for the number of inhabitants [8], we include in the dynamical description of the time evolution of the system the resources that, in general, cannot be considered as constants. A generalization of the logistic equation to an arbitrary number of homogeneous species interacting among the individuals (with non-constant resources) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}N_i = r_i N_i \left[1 - \frac{N_i}{N_{ci}\left(R_1 \cdots R_k\right)}\right] - \sum_{j=1\,, i \neq j}^{m} \chi_{ij} N_j N_i , \qquad (1)$$

where r_i is the usual growing rate for species i. In the denominator there appears the carrying capacity of the system with respect to the number of inhabitants $N_{ci}(R_1 \cdots R_k)$. Beside the dependence on the resources R_i, we could have also a dependence on another species that would be then a “resource” for some other species. This fact is expressed even by the quantities χ_{ij} that in general are not a symmetric expression ($\chi_{ij} \neq \chi_{ji}$) since the prey is a resource for the predator and not the inverse. Similarly, for the resources we have

$$\frac{\mathrm{d}}{\mathrm{d}t}R_i = r_i' R_i \left[1 - \frac{R_i}{R_{ci}}\right] - \sum_{j=1}^{m} \alpha_{ij} N_j R_i. \tag{2}$$

It is clear that the set of equations (2) could be formally included into eq. (1) redefining the quantities α_{ij}. Nevertheless, we shall keep this distinction for the sake of clarity especially referred to resources, such as trees, oil or oxygen, where the carrying capacity is not determined by other species and can be considered as a constant (R_{ci}). Also the meaning of the parameters r_i' is the same of the analogous parameters r_i. It is suitable to define r_i' as “renewability ratios”, since they describe the capacity of the resources to renew themselves and clearly are depending on the kind of resource. In general all parameters of eqs. (1) and (2) are time dependent, including stochasticity. We can assume that, for the ancient societies, the parameters are reasonably slowly time-varying so that we can consider them as constants [9], in particular the α coefficients. We can call this set of parameters *technological parameters* in the sense that they carry the information about the capacity to exploit the resources of the habitat. We shall see in the next section that the technological parameters combined with the renewability ratios will be the key point to decide whether, or not, a society is destined to collapse.

Easter Island two-variable collapse model

In general modelling a society is a tough problem. Social dynamics and stochastic processes should be taken into account [10–13]. However, the particular history of Easter Island society presents several advantages for modelling its evolution [14]. In fact it can be with very good approximation considered as a closed system. The peculiar style of life and culture allows us to consider a basic model where the trees are essentially the only kind of resources. Many of the activities of the ancient inhabitants involved the trees, from building and transport the enormous Moai, to build tools, boats for fishing, etc. In fact the cold water was not adapt to the fish life and the impervious shape of the coast made fishing difficult. Finally from historical reports it can be inferred that the inhabitants did not change the way to exploit their main resource, even when very near to exhaust it. So that we can consider the technological parameter as constant. The unknown function $N_c(R)$ has to satisfy few properties and it is clear that the choice of this relation is quite arbitrary but following the simplicity criteria we can select $N_c(R) = \beta R$, where β is a positive parameter. This choice formalizes the intuition that the maximum number of individuals tolerated by a niche is proportional to the quantity of resources. In this particular case, with two variables, we can rewrite eqs. (1) and (2) as

$$\frac{\mathrm{d}}{\mathrm{d}t}N = rN\left[1-\frac{N}{\beta R}\right], \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}R = r'R\left[1-\frac{R}{R_c}-\alpha_E N\right], \qquad \alpha_E \equiv \frac{\alpha}{r'}. \tag{4}$$

The dimensionless parameter α_E is the ratio between the technological parameter α, representing the capability of exploiting the resources, and r' the renewability parameter representing the capability of the resources to regenerate. We will call α_E the deforestation parameter, since it gives a measure of the rapidity with which the resources are going to exhaust and hence a measure of the reversibility or irreversibility of the collapse. We note that in (4) the interacting term depends on the variable R. Namely, for $R = 0$ no variation of the resource exists ($\frac{\mathrm{d}}{\mathrm{d}t}R=0$) corresponding to a biological criterion and different from that of ref. [15]. Using the historical data, we can have an estimation of the parameters. At the origin ($t = 0$) we can assume that the trees were covering the entire island surface of 160 km^2. When the first humans arrived at the island, around 400 A.C., their number was of the order of few dozens of individuals and it grew until reaching the maximum around 1300 A.C. [4, 16]. A maximum value of $N_M \sim 7000$ inhabitants is widely quoted by archeologists but other estimates range up to 20000.

Applying the standard technique to find the equilibrium points to eqs. (3) and (4), we obtain the two points:

$$N_0 = 0, \quad R_0 = R_c \quad \text{(unstable saddle point)}, \tag{5}$$

$$N_e = \frac{\beta R_c}{1+\alpha_E \beta R_c}, \quad R_e = \frac{R_c}{1+\alpha_E \beta R_c} \quad \text{(stable)}. \tag{6}$$

The point (N_0, R_0), at equilibrium, represents the trivial fact that in the absence of human beings the number of trees (or more in general the resources) goes to the saturation value. On the other hand, eq. (6) describes the fact that, due to the humans-environment interaction, the other equilibrium point, (N_e, R_e), does not coincide with (βR_c, R_c) since $\alpha \neq 0$. To study the stability of the point (N_e, R_e), we have linearized the system of equations (3) and (4) around the equilibrium point. The analytical calculations show that the linearized system has two complex eigenvalues with the real part always negative allowing us to conclude that the point (N_e, R_e) is a stable-equilibrium point.

Collapse condition

Even if, mathematically speaking, the stable point (N_e, R_e) is an acceptable result, we have to take into account the biological constraints that allow a species to survive. A reasonable number of individuals is required for viability of a given species [17]. This is so because genetic diversity, social structures, encounters, etc., need a minimum number of individuals since under this critical number the species is not viable and collapses.

Calling the minimum number of humans N_{min}, we can find an upper bound for the parameter α_E, so that a civilization can survive. Imposing the collapse condition $N_e \leqslant N_{min}$, we obtain

$$\alpha_E \geqslant \frac{1}{N_{min}} - \frac{1}{\beta R_c} \quad \text{(collapse condition).} \tag{7}$$

As further simplification of inequality (7), we assume that $N_{min} \ll \beta R_c$ and we find that $\alpha_E N_{min} \geqslant 1$ or $\alpha N_{min} \geqslant r'$ (see (4)). It is a natural condition since it tells that collapse exists when the production rate r' is smaller than the deforestation rate αN_{min}.

Deforestation rate estimation

As we saw in the previous section, the collapse condition (7) gives a sufficient condition on the deforestation rate per individual α. On other hand, the last period of the exploitation of the resources was governed essentially by the deforestation rate. In this way, from (4) we have the rate of tree extinction

$$\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} \sim -\alpha N. \tag{8}$$

As discussed, the path to the equilibrium point is exponentially fast, so that a rough estimation of the left side of eq. (8) is the time scale of the deforestation, τ_F, while the right side can be taken at the end of the collapse process:

$$\frac{1}{\tau_F} \sim \alpha N_F, \qquad (9)$$

being N_F the final number of individuals. Since it can be deduced [16] that τ_F ranges from $\tau_F \sim 100$ y to $\tau_F \sim 300$ y, while N_F ranges from $N_F \sim 2000$ to $N_F \sim 3000$, the rate of deforestation (per individual) could be estimated as

$$\alpha \sim \frac{1}{\tau_F N_F} \quad (\text{y individual})^{-1}, \qquad (10)$$

giving a predictive range

$$1.1 \times 10^{-6} < \alpha < 3.3 \times 10^{-6} \quad (\text{y individual})^{-1}. \qquad (11)$$

This estimation has validity in the case of exponential decay which is our case.

Assuming the number of to be trees proportional to the area A, we can now estimate the deforestation rate area. The island has a surface of the order of $A \sim 160$ km^2 and it can be supposed that initially it was covered with trees so that we can estimate that

$$0.5 < \frac{\mathrm{d}A}{\mathrm{d}t} \sim \frac{A_0}{\tau_F} < 1.6 \ (\text{km}^2/\text{y}). \qquad (12)$$

As a comparison we can consider that in the last 500 years the deforestation rate of the Amazonian forest is 15 (km^2/y). Considering that in 500 years the deforestation technology became more and more efficient, especially in the last century, we can consider it as an upper limit, giving us an idea of the technological change.

In the human history there are several examples of overexploiting the natural resources even if not so known as in the case of Easter Island. In particular, the Copán Maya history [18] has a certain similarity with respect to the technology level and the overexploiting of the natural resources. In short, this ancient civilization reached almost 20000 individuals and declined to 5000 individuals in the 9th century. Using the estimation of the technological parameter α obtained for the Eastern Island civilization (11), we get a collapse time from eq. (9) that is

$60 < \tau_F < 180$ years. The collapse time based on historical reports is 100 years, showing that the adopted model is consistent with the available data. This estimation is consistent with the idea that similar civilizations, in technological meaning, have a similar capacity to exploit the natural resources.

Demographic curve prediction

In this section we estimate numerically the evolution of the number of individuals of the Easter Island. For the numerical values of the parameters we use values found in the literature and predicted in the above section. The value of the technological parameter was estimated in the previous section and we use $\alpha \sim 2 \times 10^{-6}$ (y individual)$^{-1}$. As value of the growing rate we use the value of the ancient populations $r \sim 0.01$ (y)$^{-1}$ (Fort *et al.* [19]). For the palm trees we use a rate slower than human species; indeed we consider a rate $r' \sim 0.001$ (y)$^{-1}$. It is worthy to stress that being it anyway $r' \ll r$, its exact value does not affect appreciably the time evolution of $N(t)$.

Before continuing it is important to introduce the concept of effective area. So far we have considered only palms trees as technological resource; nevertheless, there were other kinds of technological resources (marine bones, shrub, etc.) partially used as valid substitutes of palms. These resources allowed the ancient inhabitants to exploit an area larger than the island area. Indeed they were shipping several kilometers far from the coast for the fishing activity. In other words, there is an "effective number of palms" (proportional to the exploited area) that includes other resources; an area that is larger than the island area. Considering the effective-area concept, we are able to take into account the other resources besides the palm trees as, for example, the fishing activity. For this purpose we consider that $R \propto A$ and we can rewrite the system of equations (3) and (4) in terms of the exploited area (or effective area). For what we said, we can assume as initial value of the effective area $A \sim 3 \times 160$ km^2 and eqs. (3), (4) become

$$\frac{\mathrm{d}}{\mathrm{d}t}N = rN\left[1-\frac{N}{\beta' A}\right], \qquad (13)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}A = r'A\left[1-\frac{A}{A_c}\right]-\alpha N A. \qquad (14)$$

Considering as maximum niche capacity a number of individuals of the order of 20000 (corresponding to the largest estimate present in the literature), the value of β' is given by $\beta' A_c \sim 20000$, where $A_c \sim 3 \times 160$ km^2.

Figures 1 and 2 show the demographic evolution of the inhabitants $N(t)$ and the evolution of the resources (effective area $A(t)$) of the Easter Island. The numerical solution, inspired by eqs. (13) and (14), is in good agreement with the archeological data. Despite its simplicity, the model gives a reasonable prediction of the demographic structure of the population in the island. Here a chronological division appears, quite standard for ancient culture. The first epoch (400–800 A.C.) is the initial growing of the population and the slow exploitation of the resources. In the intermediate period (800–1400 A.C.), the massive exploitation operates and, finally, the extinction of the resource and the collapse of the civilization (1400–1600 A.C.). This chronological division is in good agreement with the archeological conception of the island.

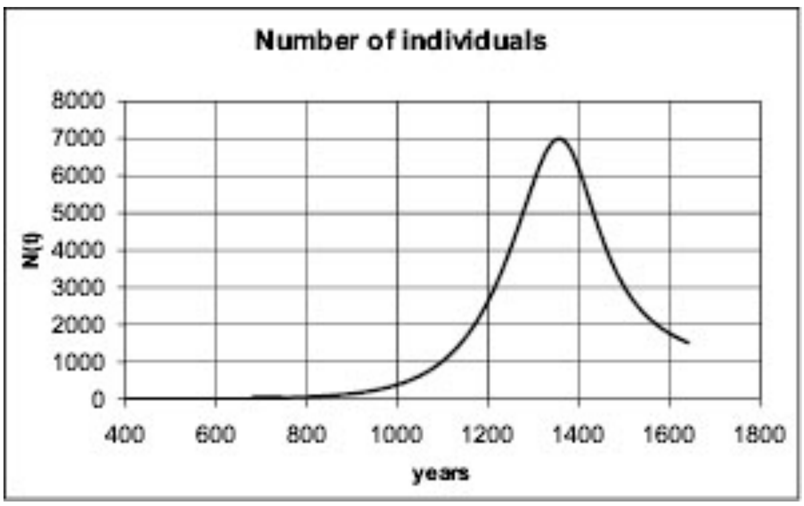

Figure 1. Number of inhabitants *vs*. time inspired by the model (13) and (14). The maximum size of the population, $N_M \approx 7000$ inhabitants, and the equilibrium size of the population, $N_F \approx 2000$, is in very good agreement with the archeological data.

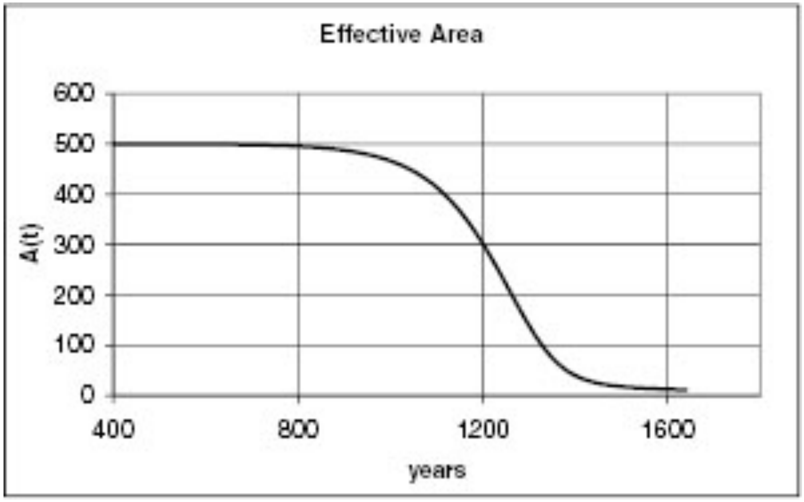

Figure 2. Resources (effective area) *vs*. time inspired by the model (13) and (14).

Concluding remarks

A mathematical model, considering the interaction between carrying capacity and population, has been considered and applied to the Easter Island society. We presented a prediction of the demographic curve (fig. 1). Moreover, an estimation of the technological parameter α is obtained and applied to another ancient civilization, the Maya of Copán, with a reasonably precise estimation about their collapse time, confirming the consistency of the adopted model.

Acknowledgments

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